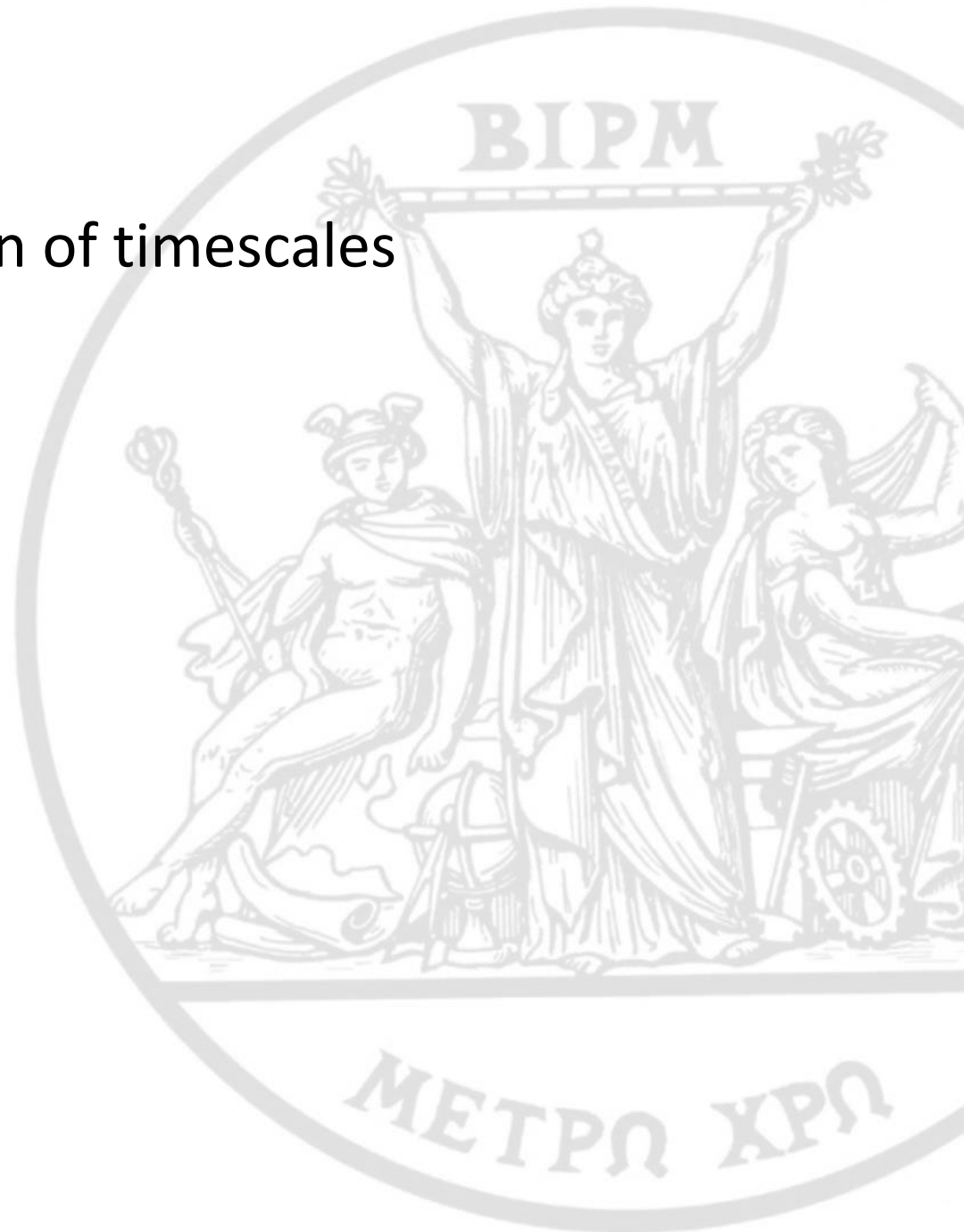


# Definition and realization of timescales and links with geodesy

Gérard Petit  
BIPM

JWG 2-1  
Hannover 15-16 May 2017

**B**ureau  
International des  
**P**oids et  
**M**esures



# IAU framework: coordinate times for the Earth

- ◆ IAU'1991:
  - Names **TCG** the time coordinate of the **geocentric system**
  - Defines **TT** another coordinate time “differing from TCG ... by a constant rate, **the unit of measurement of TT** being chosen so that **it agrees with the SI second on the geoid**”
- ◆ IAU'2000:
  - “TT be a time scale differing from TCG by a constant rate:  $dTT/dTCG = 1 - L_G$ , where  $L_G = 6.969290134 \times 10^{-10}$  is a defining constant ”
- ◆ Impact of the change from IAU'1991 to IAU'2000:
  - ◆ No more dependent on the geoid => important for space clocks
  - ◆ The change will have impact at the  $10^{-17}$  level. At the  $10^{-16}$  level, one can continue thinking in terms of "geoid".

# Transformation from proper time to TCG/TT for an Earth clock

- ◆ For a clock at rest, assuming tidal terms are negligible:

$$d\tau/dTCG = 1 - 1/c^2 [U(t,x) + v(t,x)^2/2]$$

i.e.  $d\tau / dTCG = 1 - W/c^2$

where **W is the gravity (gravitational + rotational) potential** at the clock (with the gravitational potential vanishing at infinity)

- ◆  $dTT / dTCG = 1 - L_G$  where  $L_G = 6.969290134 \times 10^{-10}$  (defining)

Equivalently, TT has the same rate as "proper time on the equipotential  $W_0 = 62636856.0005 \text{ m}^2\text{s}^{-2}$ ", i.e. **at the time of deciding the change of definition of TT** the geoid is the equipotential  $W_0 = 62636856 \text{ m}^2\text{s}^{-2}$

$$d\tau / dTT \approx 1 + (W_0 - W)/c^2 = 1 - W/c^2 + L_G$$

- ◆ Comparing two clocks  $v_B/v_A \approx 1 + (W_B - W_A)/c^2$

- ◆ On Earth:  $dW = -g \times dH$  where  $g$  is the gravity acceleration and  $H$  the geometric height  
Computing  $\Delta W = -\int g \cdot dH$  necessitates to know (measure)  $g$  and  $dH$  along the path.

# Reference timescales (1/2): Current definitions

- ◆ TAI was defined in 1970 without much attention to relativity
- ◆ TAI definition completed in 1980 with mention of the geoid
- ◆ IAU then sets the relativistic framework in Res A4 (1991)
  - TCG and TCB
  - TT differs from TCG by a constant rate; unit of measurement of TT agrees with the SI second on the geoid.
- ◆ IAU re-defines TT in B1.9 (2000) with a fixed conventional rate difference to TCG

## Recommendation S 2 (CCDS, 1970), in CIPM, 1970

### → Definition of TAI

International Atomic Time (TAI) is the time reference coordinate established by the Bureau International de l'Heure on the basis of the readings of atomic clocks operating in various establishments in accordance with the definition of the second, the unit of time of the International System of Units.

In 1980 the definition of TAI was completed as follows (declaration of the CCDS, *BIPM Com. Cons. Déf. Seconde*, 1980, **9**, S15 and *Metrologia*, 1981, **17**, 70):

TAI is a coordinate time scale defined in a geocentric reference frame with the SI second as realized on the rotating geoid as the scale unit.

### Resolution No. B1.9

*Re-definition of terrestrial time TT*  
*Redéfinition du temps terrestre TT*

#### Recommends

.. that TT be a time scale differing from TCG by a constant rate :  $dTT/dTCG = 1 - L_G$ , where  $L_G = 6.969\,290\,134 \times 10^{-10}$  is a defining constant,

#### Note

$L_G$  was defined by the IAU Resolution A4 (1991) in its Recommendation 4 as equal to  $U_G/c^2$  where  $U_G$  is the geopotential at the geoid.  $L_G$  is now used as a defining constant.

## Reference timescales (2/2): Proper time to TT

### ◆ In principle:

- Clock's proper time allows realizing TCG:  $d\tau/dTCG = 1 - 1/c^2 [U(t,x) + v(t,x)^2/2]$
- TT is derived from TCG using defining formula  $dTT/dTCG = 1 - L_G$ ;
- **That system would perfectly apply for clocks in space**

Clock  
↓  
TCG  
↓  
TT

### ◆ In practice, clocks are still on Earth

- Clock's proper time is more or less loosely corrected to the geoid, estimating for the gravity potential difference between clock and geoid.
- This defines TAI, assimilated to a realization of TT.

Clock  
↓  
TAI(geoid)  
↓  
~TT

### ◆ This is OK at the $1 \times 10^{-16}$ level or slightly below. **At the $1 \times 10^{-17}$ level we have a fundamental problem** in the definitions of timescales:

1. TT is defined from TCG using defining constant  $L_G$ ;
2. TAI is defined with respect to the geoid;
3. We expect TAI to be a realization of TT.

**These 3 statements  
are inconsistent !!**

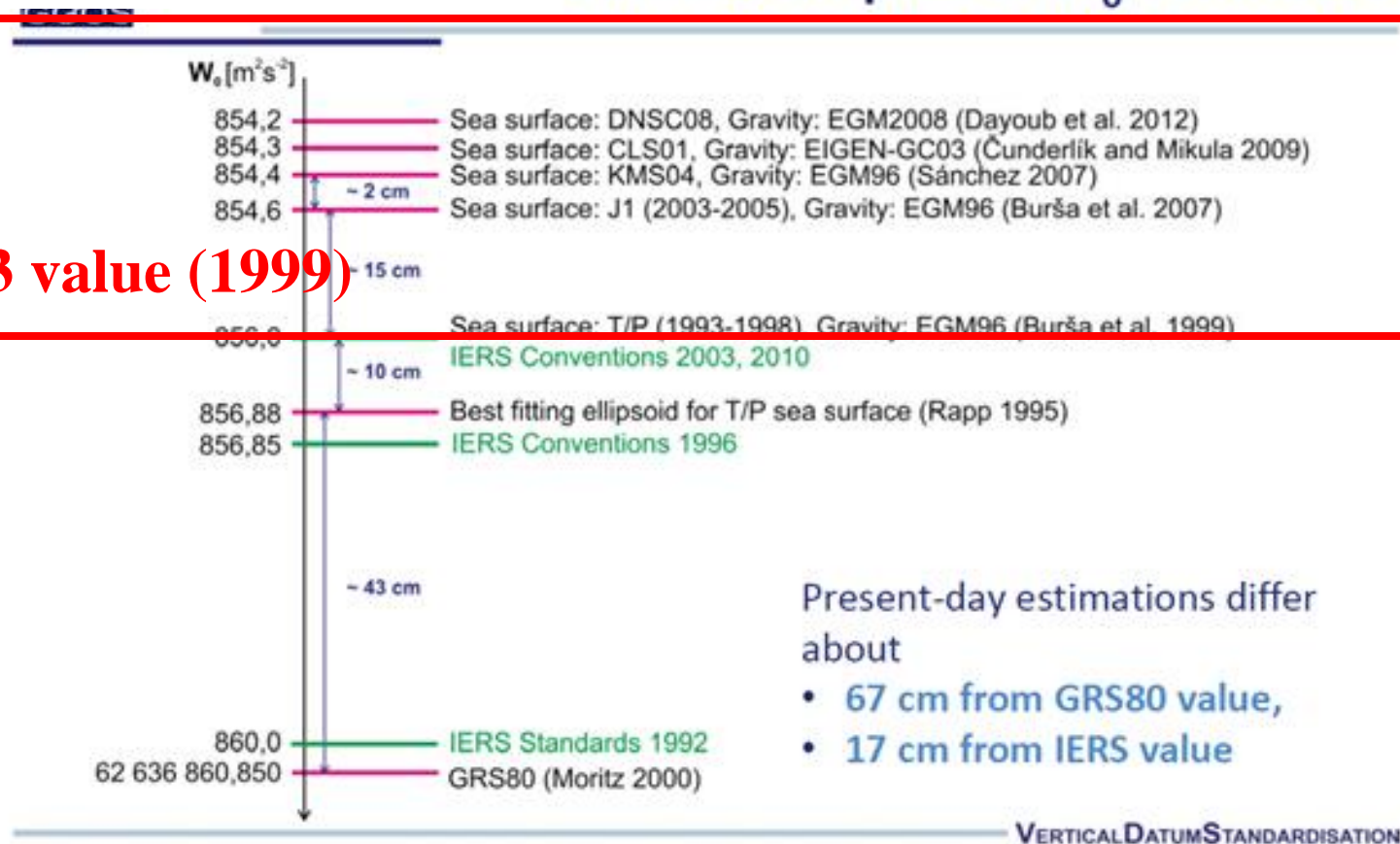
# The geoid and $W_0$ (1/2)

- ◆ The International Association for Geodesy (IAG) set-up two WGs
  - Ad-hoc Group on an International Height Reference System (IHR)
  - Joint Working Group 0.1.1 on Vertical Datum Standardization
- ◆ These WGs generated IAG Resolution 1 (2015) for the definition and realization of an International Height Reference System (IHR)
  - the following conventions for the definition of an International Height Reference System (see note 1):
    1. the vertical reference level is an equipotential surface of the Earth gravity field with the geopotential value  $W_0$  (at the geoid);
  - $W_0 = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$  as realization of the potential value of the vertical reference level for the IHR (see note 2).
- ◆ This choice lets two different values for  $W_0$ :
  - “ $W_0$  (2000)” =  $62\,636\,856 \text{ m}^2\text{s}^{-2}$  used for the definition of TT
  - “ $W_0$  (2015)” =  $62\,636\,853.4 \text{ m}^2\text{s}^{-2}$  to be used for the reference level of the IHR
  - The difference is equivalent to  $2.9 \times 10^{-17}$  in rate.

# The geoid and $W_0$ (2/2)

◆  
**IAG chosen value (2015)**

## Some examples of $W_0$ estimates



**IAG SC3 value (1999)**

# Future timescale definition?

- ◆ Are there options?
- ~~◆ Some geodesists seem to think that the definition of TT should be changed with a new realization of the geoid~~
- ◆ The geodetic community needs to update the geoid,
- ◆ However TT should not change. Then it is simpler to remove the term « geoid » from the definition of TAI.
- ◆ « TAI is a realization of TT, a coordinate time in the geocentric system, defined in IAU B1.9 (2000) Resolution. It is a continuous time scale realized by the BIPM ..... »
  - This could also be an opportunity to promote a continuous UTC.....
- ◆ Then a « mise en pratique » should provide practical solutions to the transformation of proper time (PFS) to coordinate time



# Draft Resolution proposed to the CCTF in June 2017 eventually to be transmitted to the CGPM'2018

## Recommendation on the definition of time-scales

The Consultative Committee for Time and Frequency (CCTF),

.....

**clarifies that**

- TAI is a realization of TT as defined by IAU Resolution B1.9 (2000); it is a continuous time-scale produced by the BIPM based on the best realizations of the SI second;
- in the transformation from proper time to TAI, the relativistic rate shift is computed with respect to the equipotential  $W_0 = 62636856 \text{ m}^2\text{s}^{-2}$  of the Earth's gravity potential, which conforms to the constant  $L_G$  defining the rate of TT;

.....

**recommends the following definitions of TAI and UTC**

- International Atomic Time (TAI) is a realization of Terrestrial Time (TT) as defined by the IAU Resolution B1.9 (2000) with  $TT - TAI = 32.184 \text{ s}$  exactly at 1977 January 1, 0h TAI; TAI is a continuous time-scale produced by the BIPM based on the best realizations of the SI second;
- Coordinated Universal Time (UTC) is a time-scale produced by the BIPM, based on TAI. UTC has the same rate as TAI, but differs from TAI by an integral number of seconds. The procedure for UTC adjustment is described in an ITU Recommendation. The difference TAI-UTC is published by the BIPM.

# How to obtain the gravity potential at the clock?

- ◆ Obtain the gravity potential  $W$  at a marker close to the clock.  
Three solutions:
  1. Use a **G**lobal **G**eopotential model (marker position is obtained by GPS) (**GG**)
  2. Use a **L**ocal **G**eoid (**LG**)
  3. Use a **L**evelling **N**etwork (**LN**)
  
- ◆ Then go from the marker to the atoms:
  - Standard levelling OK at short distance
  - Don't forget to account for the motion of atoms (if any)

## The situation for present (2016) primary standards (1/2)

- ◆ A purely bibliographic study of the refereed publications for some of the best primary frequency standards (Cs fountains) presently operating: estimations of the uncertainty of the relativistic rate shift range from **very conservative** to **may be somewhat optimistic**.
- ◆ In general it is not possible to find all details in such a simple bibliographic study (except for NIST).

Standard	$u_B \text{ unc} / 10^{-16}$	$u_B(\text{Rel})/10^{-16}$	Method	Ref( $u_B$ )	Ref(Relat shift)
SYRTE-FO2	2.5 to 2.9	1.	LN	Guena et al IEEE Trans 2012	TBC
NIST-F2	1.5	0.3	GG, LG, LN	Heavner et al Metrologia 2014	Pavlis and Weiss, Metrologia, 2003
NPL-CSF2	2.0 to 2.7	0.5	LN	Szymaniec et al Metrologia 2010	Riedel B 2005 priv. comm.
IT-CsF2	1.7 to 2.5	0.1	LG	Levi et al Metrologia 2014	Calonico et al Metrologia 2007
PTB-CSF2 2015	2.8 to 3.5	0.06	LG (EGG97)	Weyers et al Metrologia 2012	TBC (shift=85.67x10 <sup>-16</sup> )
PTB-CSF2 2016	2.0	0.3	LG (TBC)	idem	ITOC (shift=85.45x10 <sup>-16</sup> )
SU-CsFO2	2.5	0.5	LN (Baltic)	Domnin et al. Meas. Tech. 2013	TBC

## The situation for present primary standards (2/2)

---

- ◆ Main questions when estimating the relativistic rate shift, for the Local Geoid (LG) and Levelling Network (LN) methods:
  - How is  $W(\text{LG reference})$  set and linked to a global geopotential model?
  - What is the uncertainty?
  - Some info can probably be found in references describing the Local Geoid
  
  - How is  $W(\text{LN reference})$  set? Is it linked to a global geopotential model?
  - What is the uncertainty?
  
- ◆ A “mise en pratique” is needed ...

# Conclusions

---

- ◆ Present definition of timescales TT/TAI inconsistent at the  $10^{-17}$  level
  - Should be revisited, e.g. by stating that TAI is a realization of TT
- ◆ Easy to state a new definition, does not makes it easy to realize it.
- ◆ Computing the relativistic shift with uncertainty at least in the low  $10^{-17}$  is now realized for comparing PFS worldwide and for realizing TT
  - Much better uncertainty possible when goal is to compare PFS in the same region
- ◆ Mise en pratique needed to go below the low  $10^{-17}$  worldwide
  - Not fully necessary for present PFS, but needed for SFS / future PFS
- ◆ Relations between Geodesy and T/F metrology to be enhanced