Ultra-precise space-borne clocks for observing Earth's gravity

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High Performance Clocks and Gravity Field Determination

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Abstract Time measured by an ideal clock crucially depends on the gravitational potential and velocity of the clock according to general relativity. Technological advances in manufacturing high-precision atomic clocks have rapidly improved their accuracy and stability over the last decade that approached the level of 10^{-18}. This notable achievement along with the direct sensitivity of clocks to the strength of the gravitational field make them practically important for various geodetic applications that are addressed in the present paper.

Based on a fully relativistic description of the background gravitational physics, we discuss the impact of those highly-precise clocks on the realization of reference frames and time scales used in geodesy. We discuss the current definitions of basic

User Community Workshop 2007 Noordwijk, NL

User Community Workshop 2009 Graz, AT

Main outcome:

- Short term: full exploitation of GRACE and GOCE, continue GRACE with GRACE-FO

- Medium term: Next-Generation Gravity Mission (NGGM) (constellations with low-low SST, i.e. GRACE-2 and NGGM)

- Long term: study alternative technologies, e.g. cold-atom interferometry, clocks

- Move forward from science demonstration concepts towards sustained observations of mass transport in the Earth system supporting relevant application areas (like hydrology at catchment level)

Courtesy R. Haagmans (ESA)

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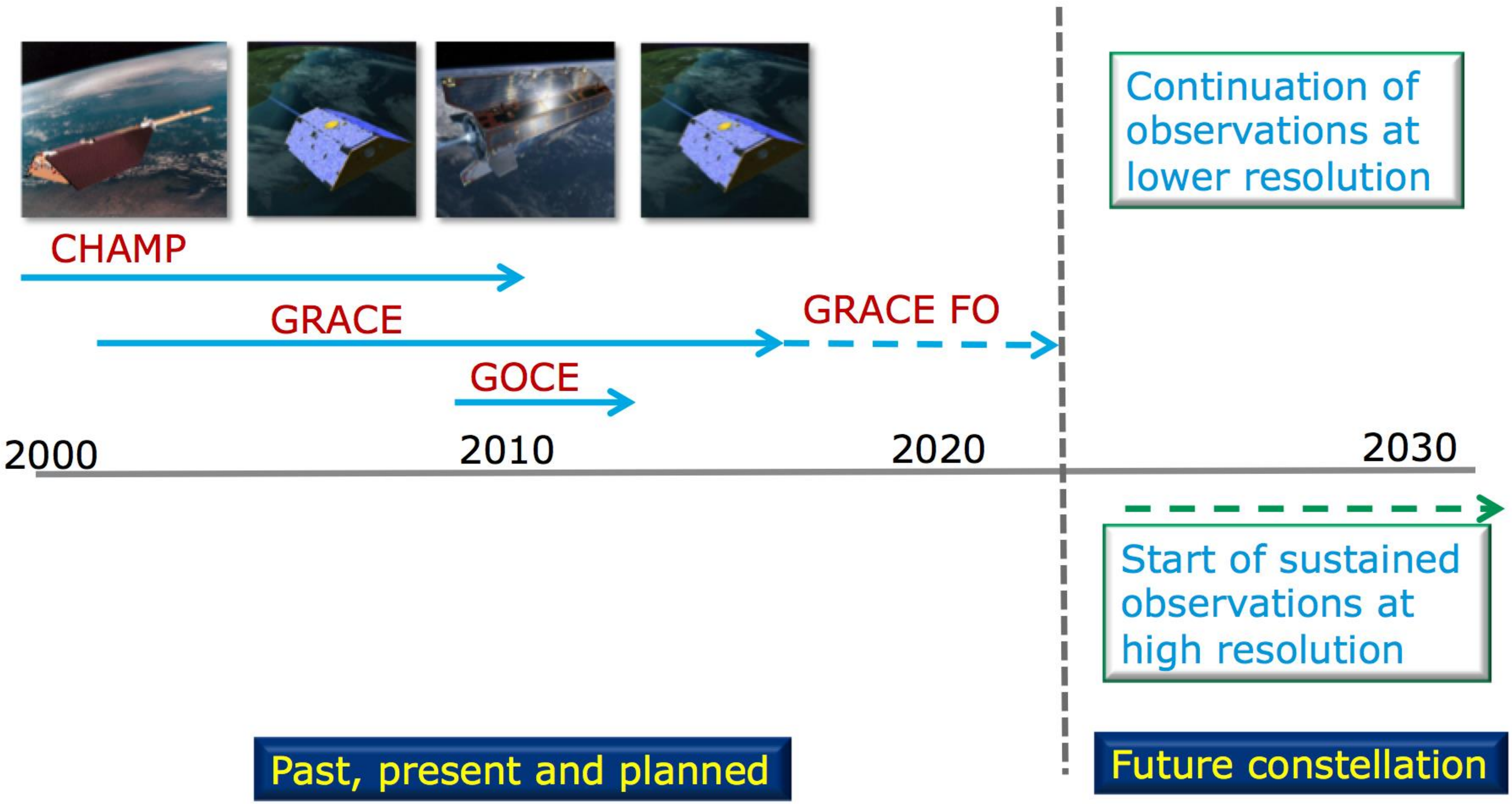
Past, present and "near" future ?



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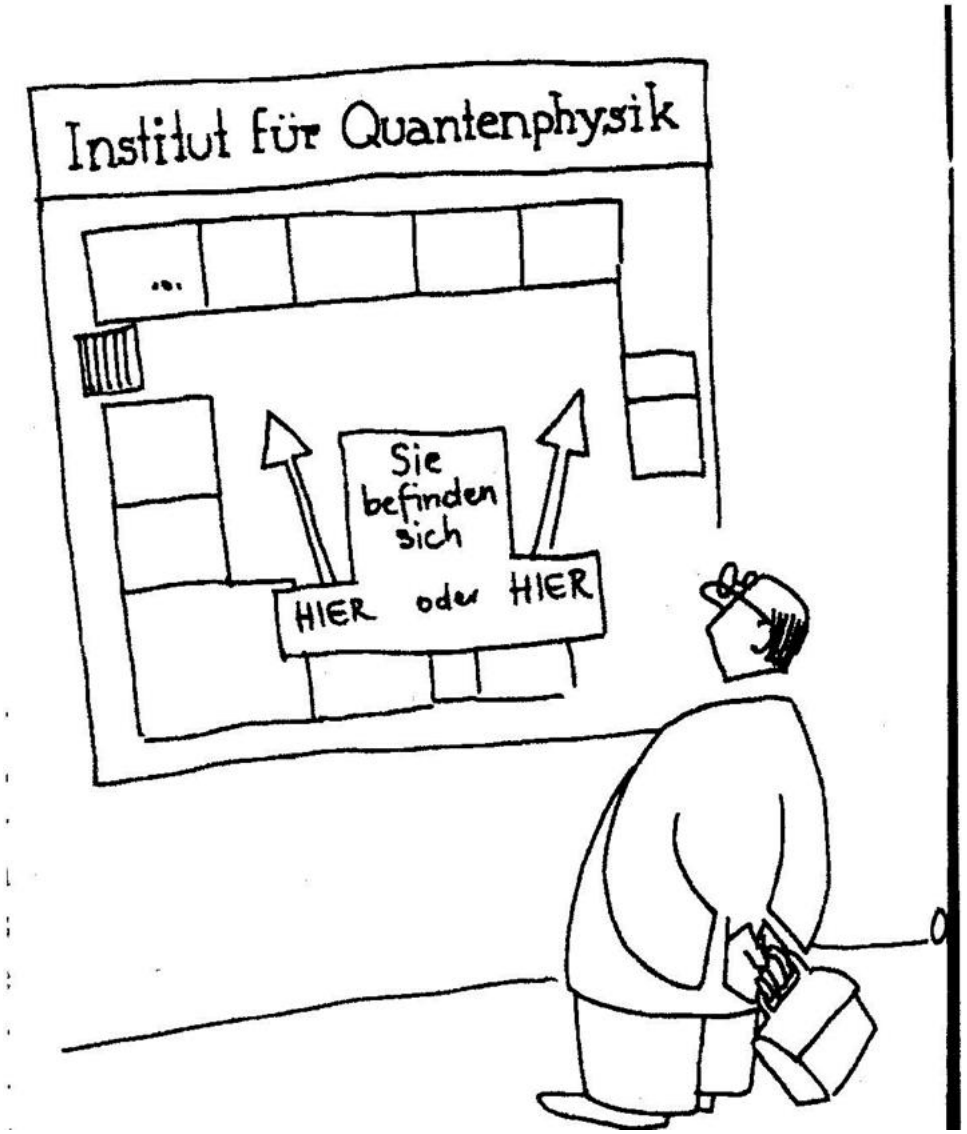
If atomic clocks reach relative accuracies of 10^{-18}:

- Measure direct potential differences with clocks (Einstein)
- Replace spirit leveling by clocks with “cm accuracy” or better
- Today we are around 3 to 5 10^{-18} !

Harrison 1 first sea chronometer

10^{-18}~1 cm in height



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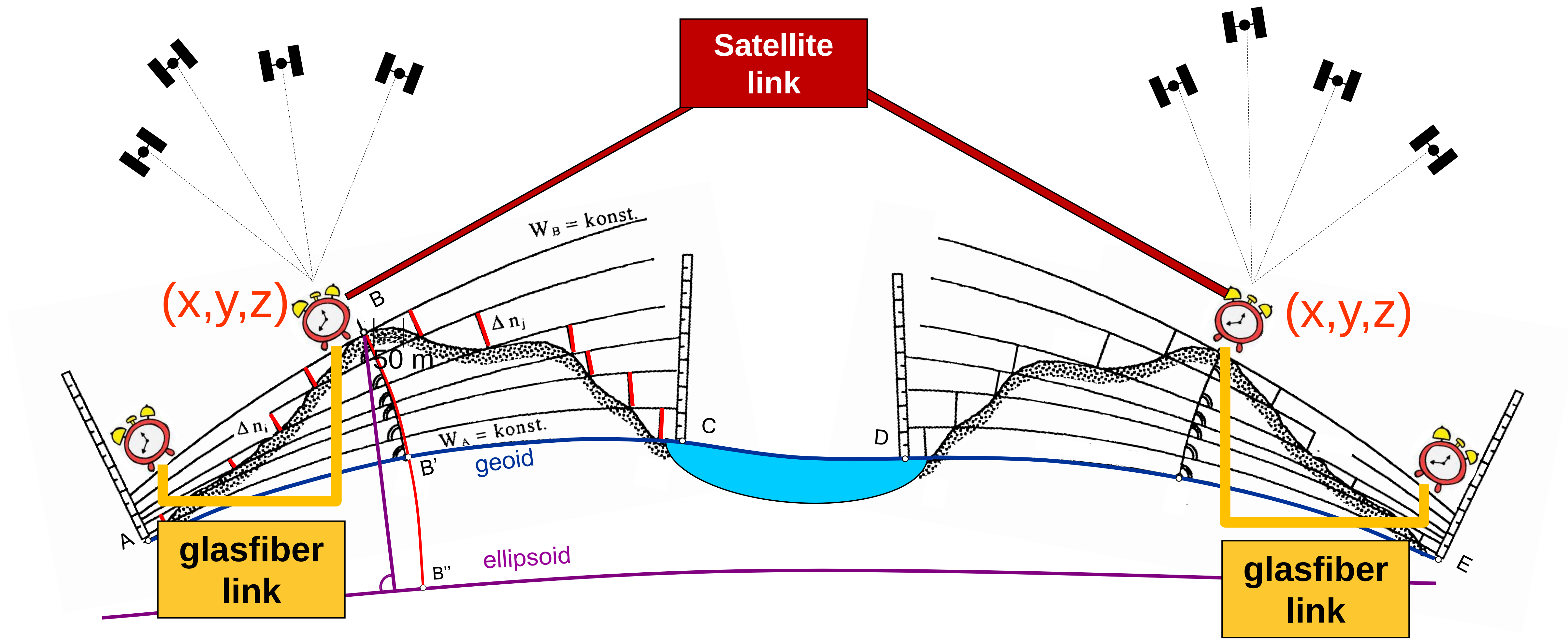
Relativistic Geodesy: unify geometry (GNSS) & gravitational positioning

Geometry measured with GPS

Gravitational potential measured with optical clocks & two-way links



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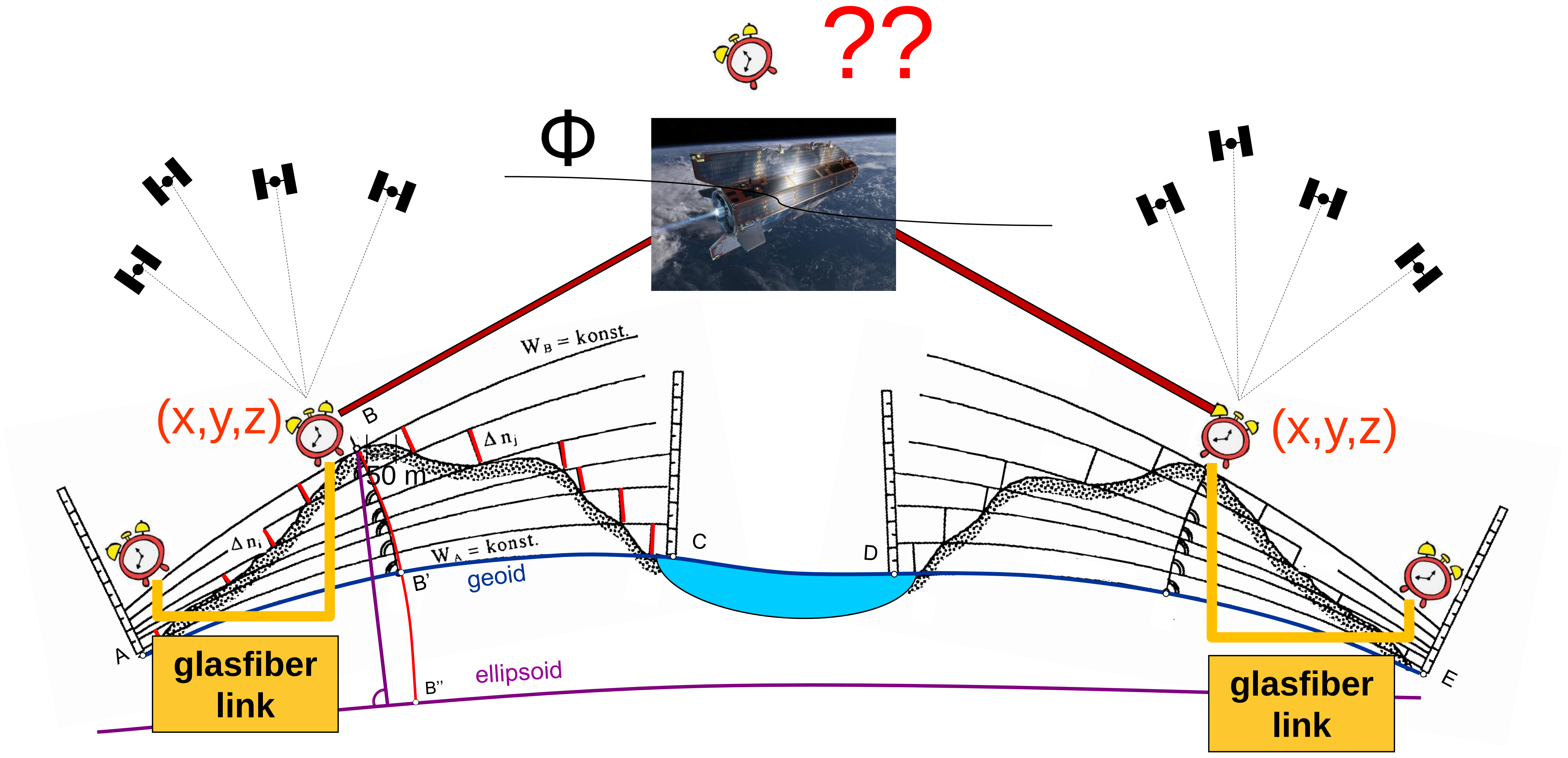
Space-borne clocks for space-borne gravimetry



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Fundamental equations

$$
{dt_E}^2 = \left(1 - \frac{2GM_i}{r_i c^2}\right) {dt_c}^2 - \left(1 - \frac{2GM_i}{r_i c^2}\right)^{-1} \frac{dx^2 + dy^2 + dz^2}{c^2}
$$

$$
v^2 = \frac{dx^2 + dy^2 + dz^2}{{dt_c}^2}
$$

$$
\frac{dt_E}{dt_c} = \sqrt{1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2}}
$$

$$
\Phi = -\frac{GM_i}{r_i}
$$

$$
\Phi = \left[\left(\frac{dt_E}{dt_c}\right)^2 - 1 + \frac{v^2}{c^2}\right] \frac{c^2}{2}
$$

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Einstein, 1911&1916 Moyer, 1981 Ashby, 200

Fundamental equations (Cont'd)

$$\Delta\Phi_{\sigma(clock)} = \left[\left(\frac{dt_E + \Delta dt_E}{dt_c}\right)^2 - 1\right]\frac{c^2}{2} \approx \frac{c^2 \Delta dt_E}{dt_E}$$

$$\Delta\Phi_{\sigma(vel)} = \left[\frac{(v+\Delta v)^2}{c^2} - \frac{v^2}{c^2}\right]\frac{c^2}{2} \approx v\Delta v$$

Fundamental equations (Cont'd)

$$\frac{\nu_R}{\nu_T} = \frac{dt_E}{dt_c} = \sqrt{\frac{c - v_{RT}}{c + v_{RT}}}$$

$$\Delta \left(\frac{dt_E}{dt_c} \right)^2 \approx \frac{2\Delta v_{RT}}{c}$$

$$\Delta\Phi_{\sigma(Doppler)} \approx \frac{2\Delta v_{RT}}{c} \frac{c^2}{2} = \Delta v_{RT} \times c$$

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Space-borne: orders of magnitude

$$g = 9 \text{ m/s}^2$$

$$c = 3 \times 10^8 \text{m/s}$$

$$v = 7.5 \times 10^3 \text{ m/s}$$

$$\frac{\Delta\Phi}{g} = 1 \text{ cm}$$

$$\Delta\Phi_{\sigma(clock)} = \frac{c^2 \Delta dt_E}{dt_E} \rightarrow \frac{\Delta dt_E}{dt_E} \approx 10^{-18}$$

$$\Delta\Phi_{\sigma(vel)} = v\Delta v \rightarrow \Delta v \approx 12 \times 10^{-6} \text{ m/s}$$

$$\Delta\Phi_{\sigma(Doppler)} = \Delta v_{RT} \times c \rightarrow \Delta v_{RT} \approx 3 \times 10^{-10} \text{ m/s}$$

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Error propagation: 1cm @ 1mHz

$$\frac{\Delta\Phi_{\sigma(clock)}}{g} \approx 30 \text{ cm}/\sqrt{\tau}$$

$$\Delta v \;=\; 360 \;\mu\text{m/s}/\sqrt{\tau}$$

$$\frac{\Delta\Phi}{g} \;=\; \frac{1}{g}\frac{\mu}{r}\{1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{a_e}{r}\right)^l (\Delta\bar{C}_{lm}\cos m\lambda + \Delta\bar{S}_{lm}\sin m\lambda)\bar{P}_{lm}(\sin\phi)\}$$

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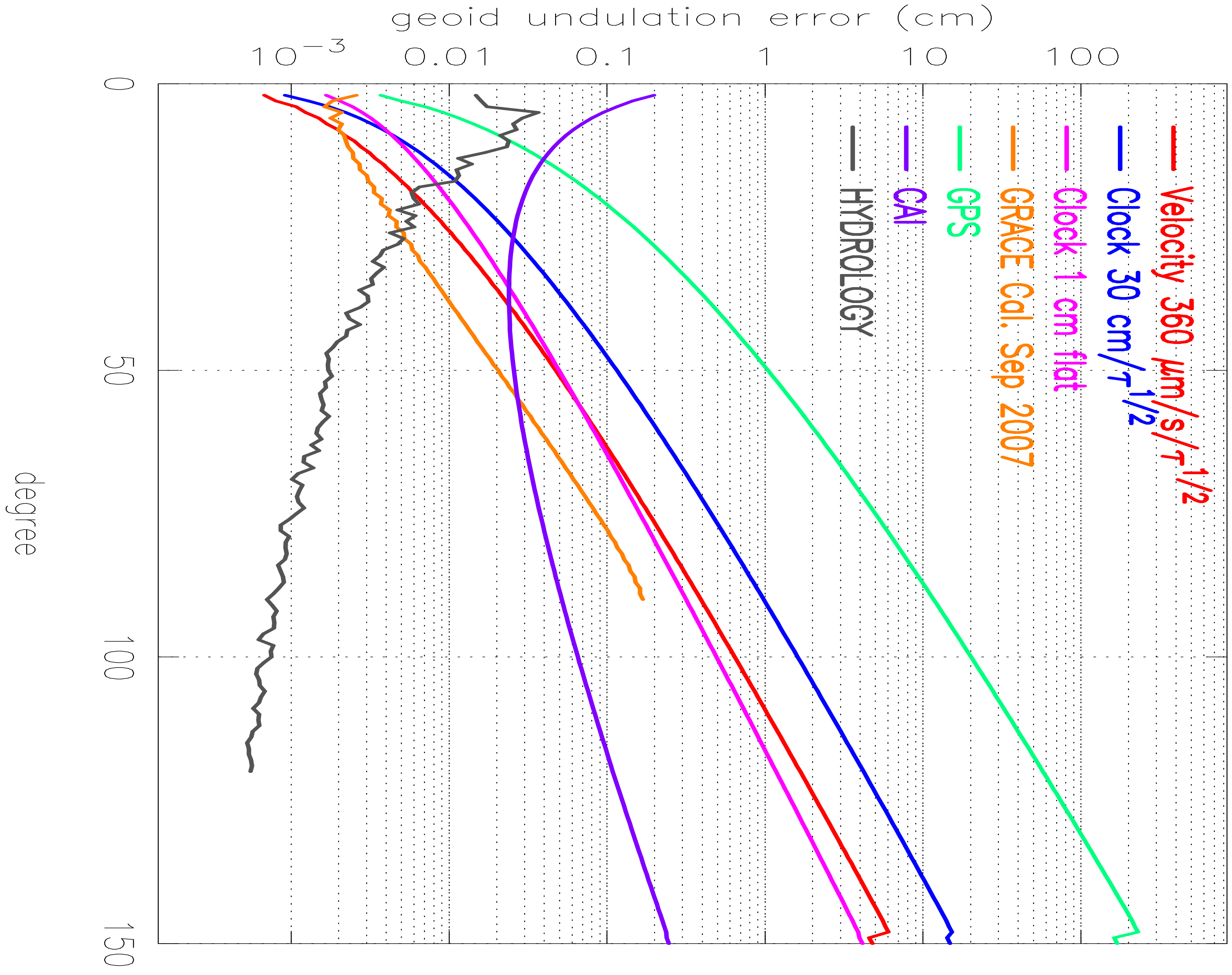
Error propagation (Cont'd)



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Conclusions

Use of space-borne clocks for global gravimetry:

- Many challenges to be met, e.g. short-term stability, reference clocks, two-way links, …
- Supporting information requited at very high level of precision, e.g. satellite velocity → can be used directly for gravimetry
- Possibly very precise frequency transfer can be used to improve the velocity determination of the satellite
- Can not compete with e.g. current GRACE ll-SST, where ll-SST technique is expected to evolve to even better precisions

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