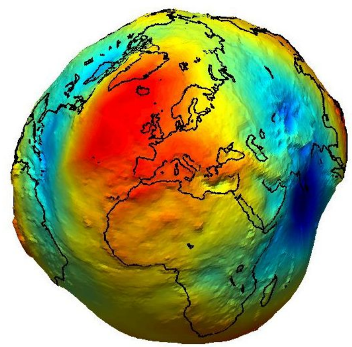
Determination of the geopotential at hight spatial resolution with optical clocks



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IAG JWG 2.1 - Relativistic Geodesy
BIPM 2018

I. Motivations

II. Contribution of atomic clocks for the geopotential determination

III. Optimization of the atomic clocks location by genetic algorithms

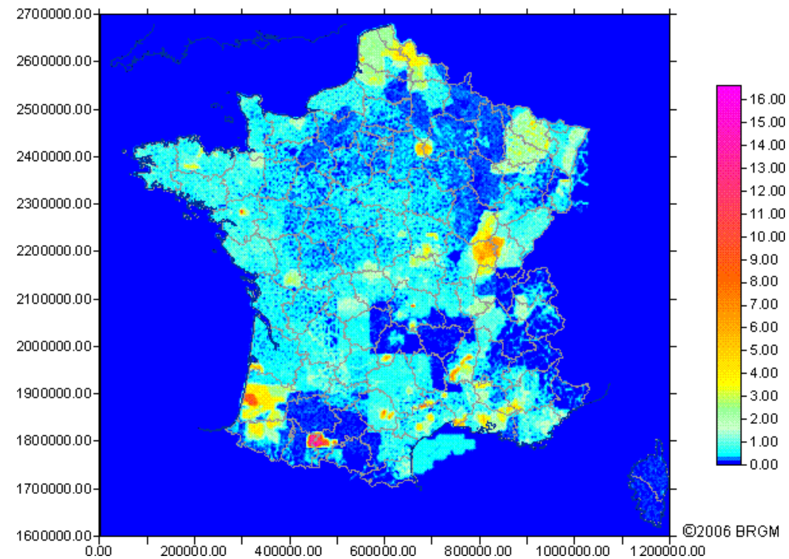
IV. Conclusions and future prospects

Problems in geodesy…

Gravity field and geopotential models

- Quality and heterogeneous coverage of data
  - Quality of the gravity field
  → 0.5 mGal (plains) - 3 mGal (mountains)
  - Quality of the geoid
  → 2 cm (plains) - less than 10 cm (mountains)
- Computation methodology (many parameters)

Distribution of gravimetric data in France
(Number of stations/km²)



Definition of vertical frames

- Many and different height systems
- Discrepancy between techniques
- Way to get geopotential heights (by computation…)

New perspectives to overcome these limitations ?

- Geodesy based on optical atomic clocks
- Nice idea to link geopotential and heights to an atomic reference

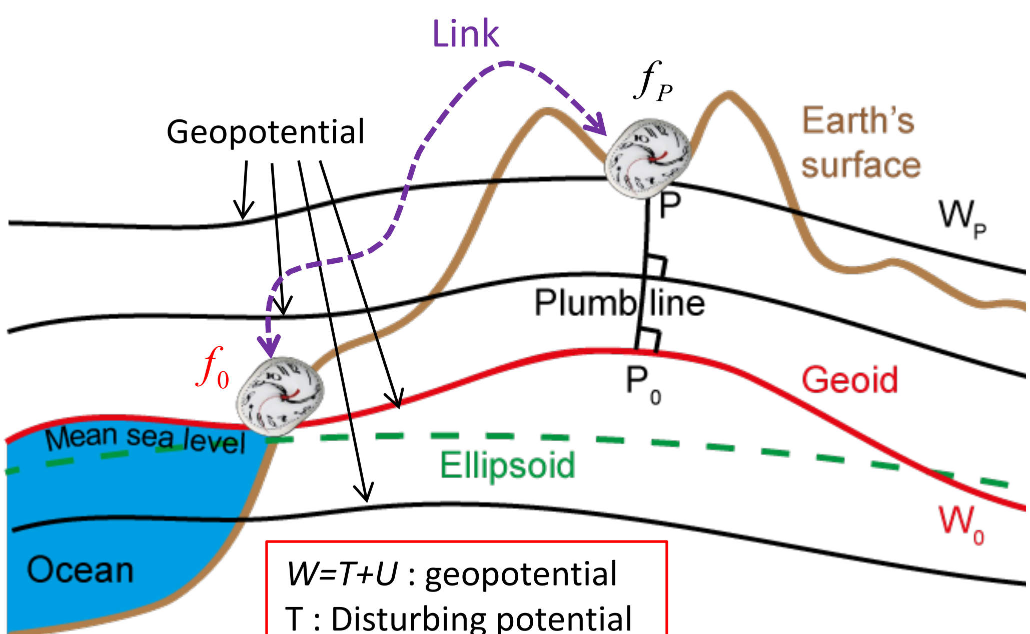
→

- Better models of geopotential and gravity field
- Better vertical reference frames

Motivations

Chronometric geodesy

- GR says that time flows differs for 2 clocks put into different gravitational potential (dilation of gravitational time)
- Clock comparisons permit to determine directly geopotential differences ΔW at the Earth's surface



W=T+U : geopotential
T : Disturbing potential
U : Normal potential
f : clock frequency

- Relativistic gravitational redshift:

$$\frac{\Delta W}{c^2} = \frac{W_P - W_0}{c^2} \approx \frac{\Delta f}{f}$$

- Related to height difference:

$$\Delta W = \frac{\mu_{Earth}}{R_{Earth}} - \frac{\mu_{Earth}}{R_{Earth} + \Delta h} \approx \frac{\mu_{Earth}}{R_{Earth}^2} \Delta h$$

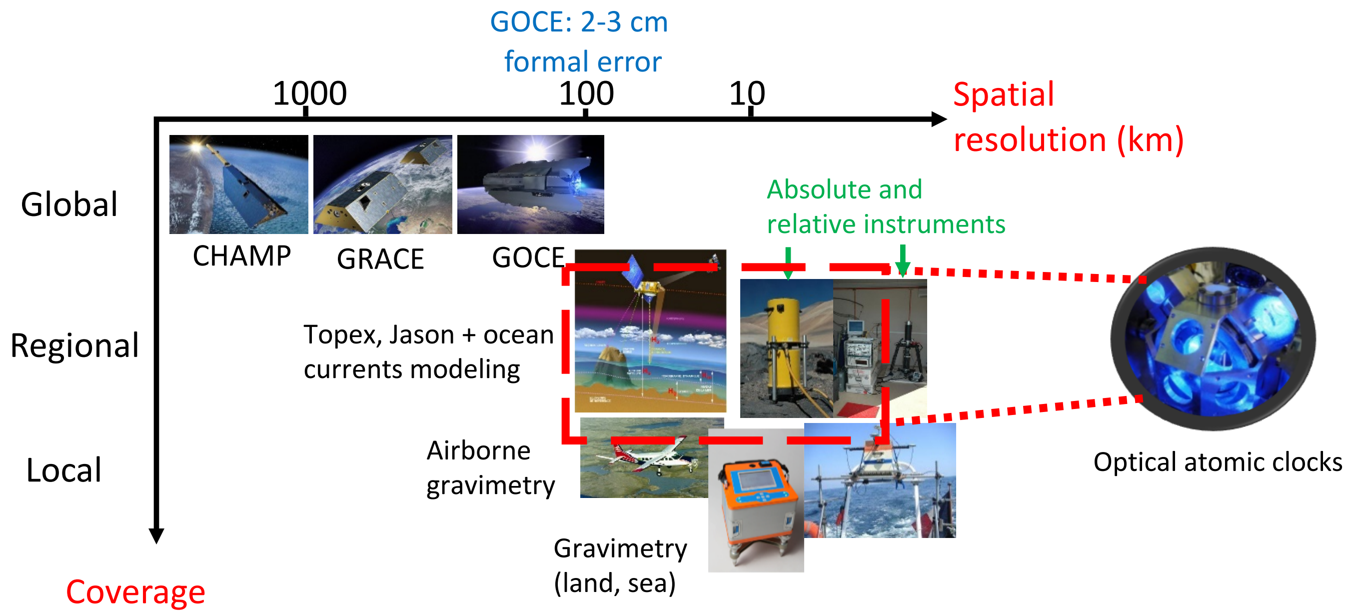
- Exactitude $\Delta f / f \approx 10^{-18}$

$$\Delta h \approx 1\ \mathrm{cm}$$

$$\Delta W \approx 0.1\ \mathrm{m^2 s^{-2}}$$

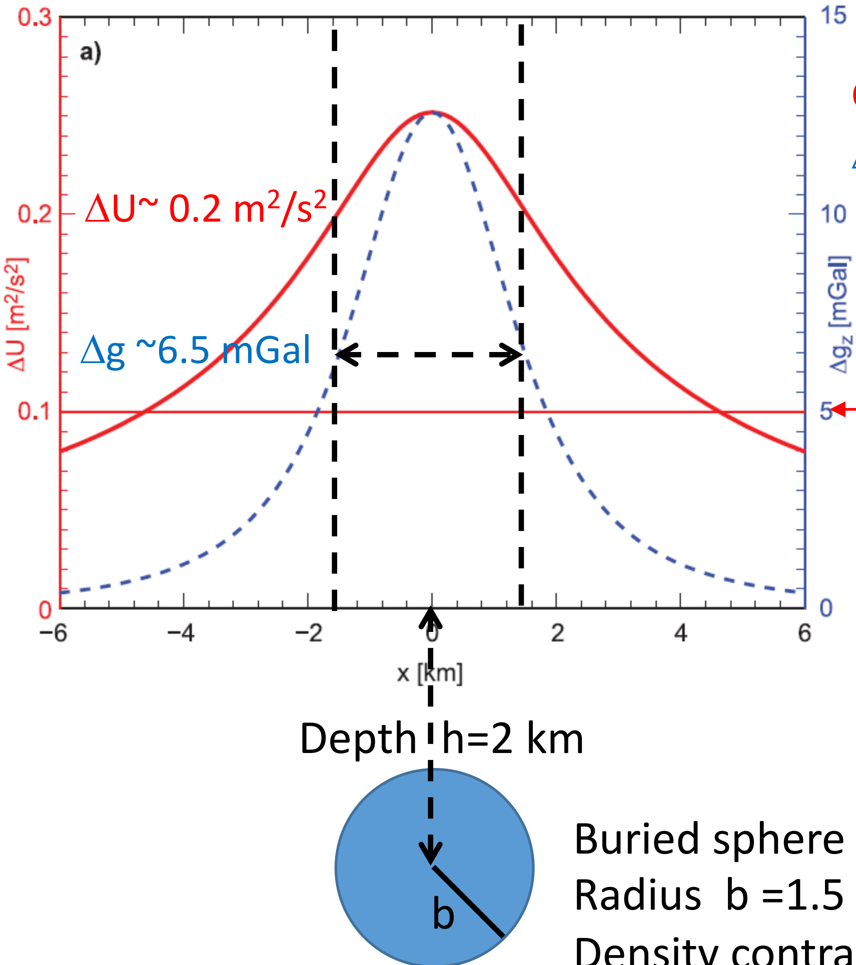
Chronometric observables in geodesy

- New type of geodetic observable → Gravitational potential differences
- Complementary to gravity and gravity gradients
- Sensitive to mass variations, including at depth
- Better spatial resolution than satellite techniques
- Reduction of heterogeneities in coverage of ground measurements
- Accuracy closer and closer to those of other techniques



- Geophysical application: gravity and geopotential signal

→ Bondarescu et al. (2012) *Geophysical applicability of atomic clocks: direct continental geoid mapping*, JGI



Geopotential anomaly ΔU and gravity anomaly Δg of a buried sphere

Detection threshold of a clock (1 cm geoid)

Buried sphere (mass M)
Radius b =1.5 km
Density contrast: $\Delta\rho/\rho$=20 %

Aims of the project

→ Evaluate the contribution of atomic clocks for the determination of the geopotential at high spatial resolution (10 km of resolution)

Study published → Lion G., Panet I., Delva P., Wolf P., Guerlin C., & Bize S. (2017): *Determination of a high spatial resolution geopotential model using atomic clock comparisons*. Journal of Geodesy

→ How to choose a good coverage of clock measurement points?

Assumptions

→ The work is based on an existing gravity network

→ All the geodetic data are synthetics, only their distribution is realistic

→ The gravity field is static and isotropic

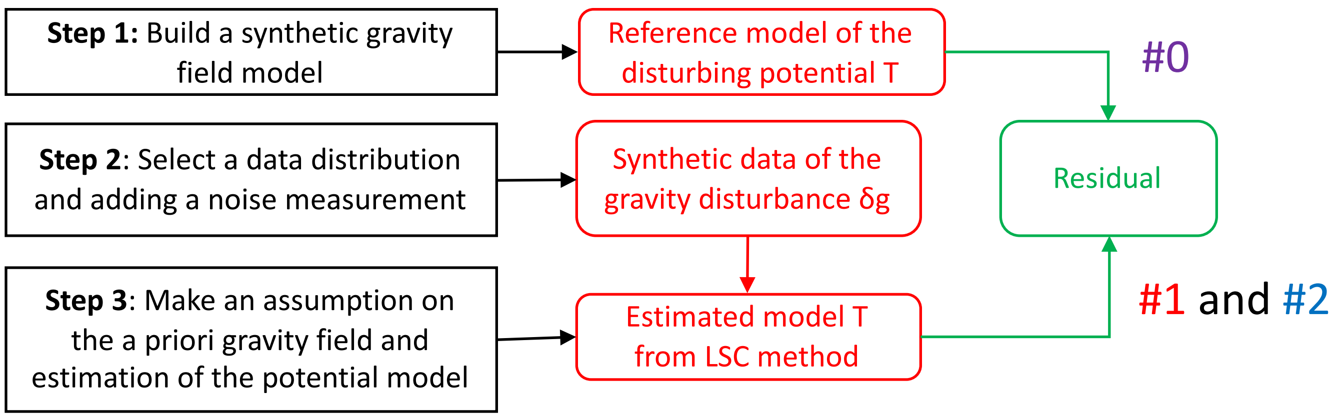
→ We work directly with the disturbing potential $T = W - U$

→ Errors on the vertical position is not considered

How to estimate the contribution of clocks data from an gravimetric network ?

- Methodology

Tools: Generation, analysis and estimation of a gravity field model



- Evaluation of the contribution of clock measurements by comparing the solutions #1 and #2 wrt a reference solution #0
  - #1: only from gravity data
  - #2: from gravity and potential data
- T is estimated on a regular grid interval of 10 km

- Estimation of the potentiel T on a regular grid using the Least Squares Collocation method (LSC) [Moritz, 1980]

Estimation of T at point P

Signal (data)

$$\tilde{T}_P = C^{T}_{T_P,l} \cdot \left(C_{l,l} + C_{\varepsilon\varepsilon}\right)^{-1} \cdot \vec{l}$$

Covariance matrix between the estimated signal and the signal

Covariance matrix of the signal

Covariance matrix of the noise

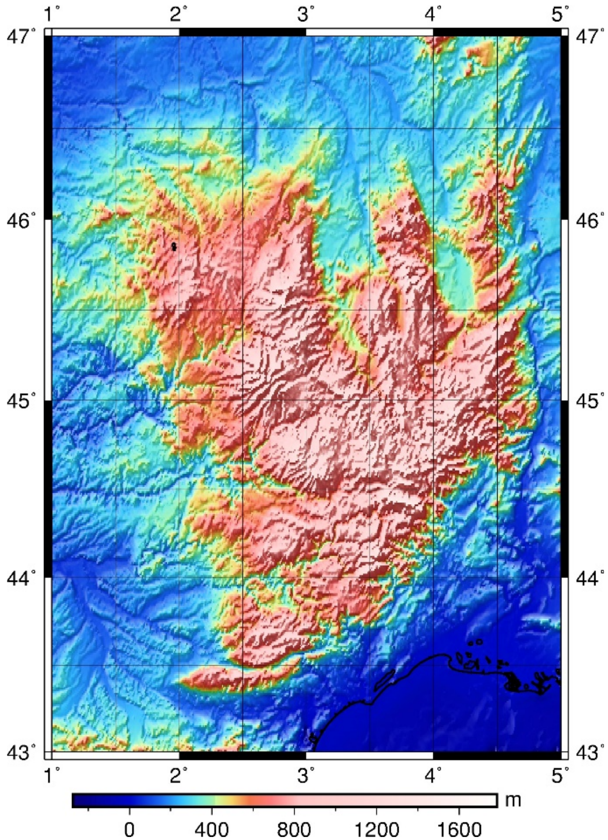
- Assumption on the gravity field a priori regularity

  → Logarithmic 3D covariance function [*Forsberg, 1987*]

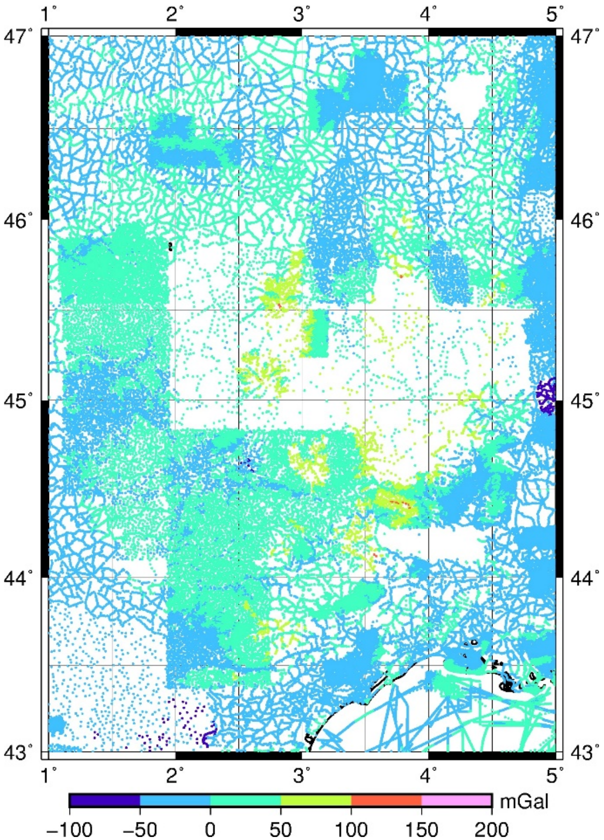
- Inverse covariance matrix is computed using a Cholesky decomposition

Massif central

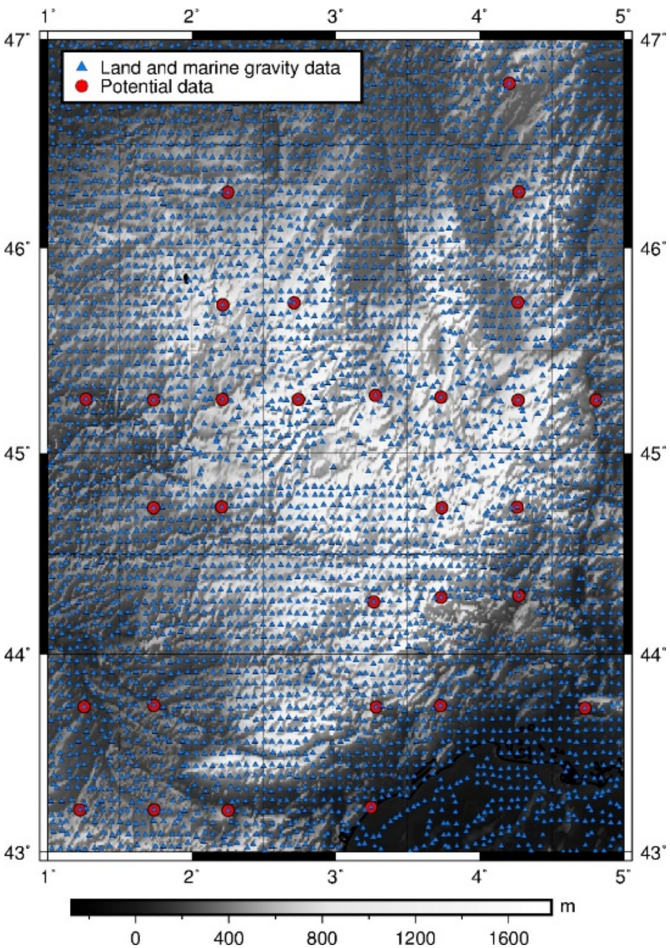
- Moderately mountainous terrain
- Intermediate gravity data coverage: 149522 data (BGI)



Topography



Terrestrial and marine free-air gravity anomalies



- 4374 reduced gravity data δg
  → noise = 1mGal
- 33 potential data T
  → noise = 0.1 m^2/s^2

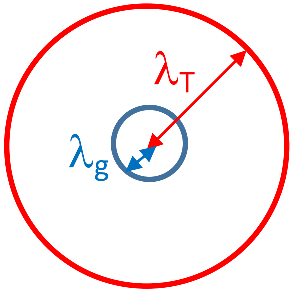
How to select the gravi points ?

→ Data reduction from the ~150000 locations
→ Distance between each point ~6.5 km
→ Each point is weighted (number of real points in the vicinity)

How to select the clock points ?

→ T more sensitive to medium wavelengths λ than δg

→ The location of the clock points is chosen to better complete the gravity network

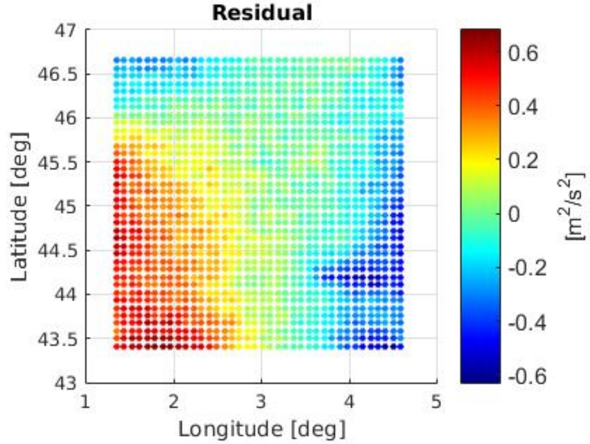


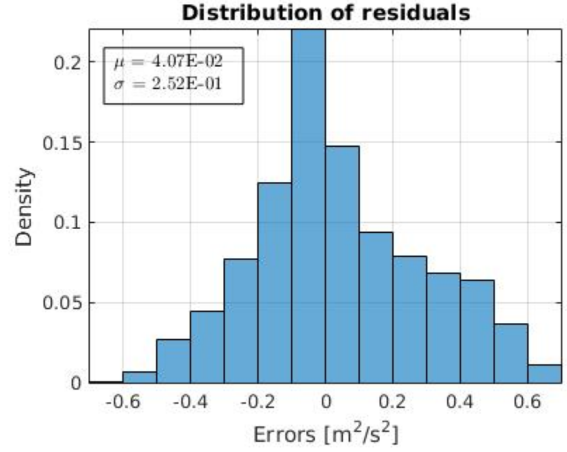
→T at same location as δg

→ Red points are an example of "handmade coverage" (not optimized)

White noise is added to the perfect synthetic data

Reconstruction of T only from data δg

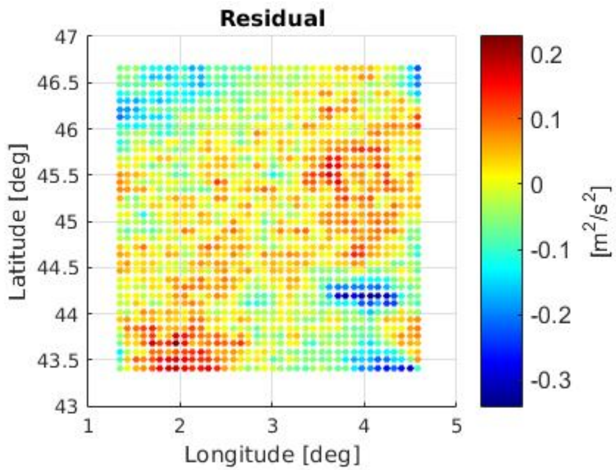


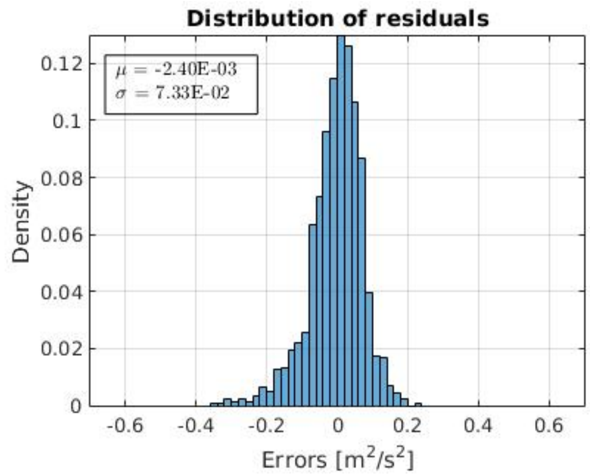


Bias: -0.04 m^2/s^2
→ 4mm

RMS: 0.3 m^2/s^2
→ 3cm

Reconstruction of T from data δg + T





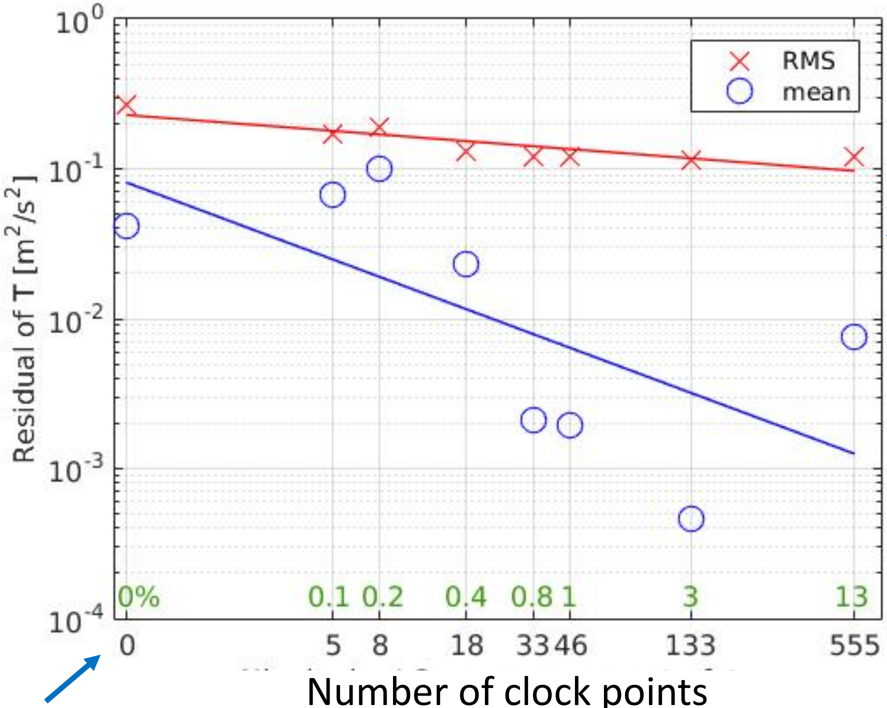
Bias: -0.002 m^2/s^2
→ 0.2mm

RMS: 0.1 m^2/s^2
→ 1cm

- **Reduction of the bias**
- **Tighter dispersion**

Results from *Lion et al. (2017)*

Effect of the clock coverage



Results from *Lion et al. (2017)*

← 1 cm

Around 30 clock points permit to reduce the bias by 1 order of magnitude

← $\frac{\text{Nb clock points}}{\text{Nb gravi points}}$ X 100

Only gravity data

Effect of the noise levels (m^2/s^2)

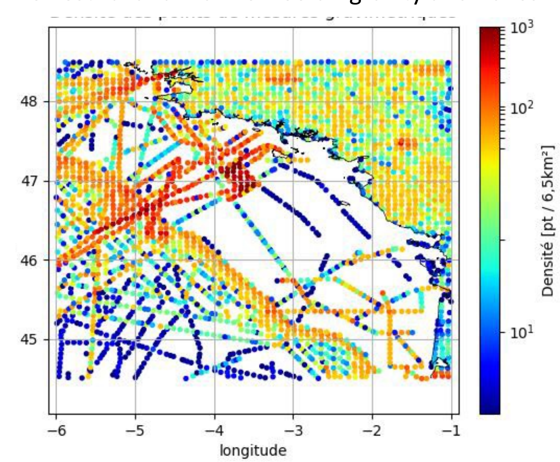
$\sigma_{\delta g}$ / σ_T	1 mGal		0.1 mGal		0.01 mGal	
	μ	σ	μ	σ	μ	σ
No clock	2.2×10^{-1}	3.9×10^{-1}	2.1×10^{-1}	4.2×10^{-1}	2.1×10^{-1}	4.2×10^{-1}
$1\ m^2\,s^{-2}$	1.4×10^{-1}	3.4×10^{-1}	1.2×10^{-1}	3.3×10^{-1}	1.2×10^{-1}	3.3×10^{-1}
$0.1\ m^2\,s^{-2}$	6.8×10^{-2}	1.7×10^{-1}	4.7×10^{-2}	1.5×10^{-1}	1.7×10^{-2}	1.6×10^{-1}

Contribution of atomic clocks for geopotential determination

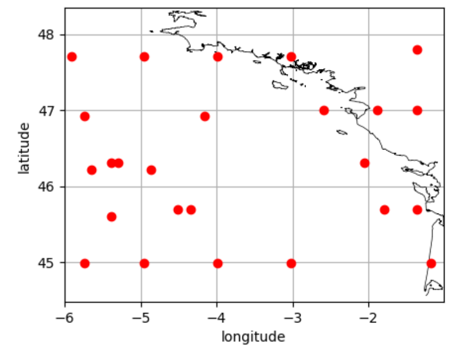
- Test of our method on the littoral zone (Atlantic coast)

→ Hugo Lecomte (master internship)

Terrestrial and marine free-air gravity anomalies



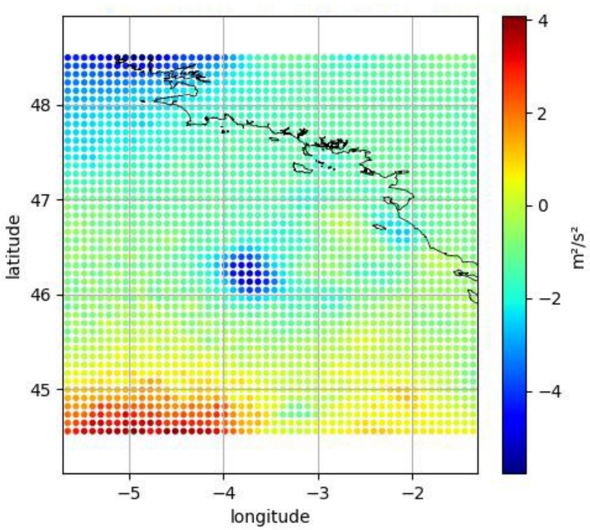
Clock coverage



Mono-covariance
Residual of T …

Multi-covariance
Residual of T …

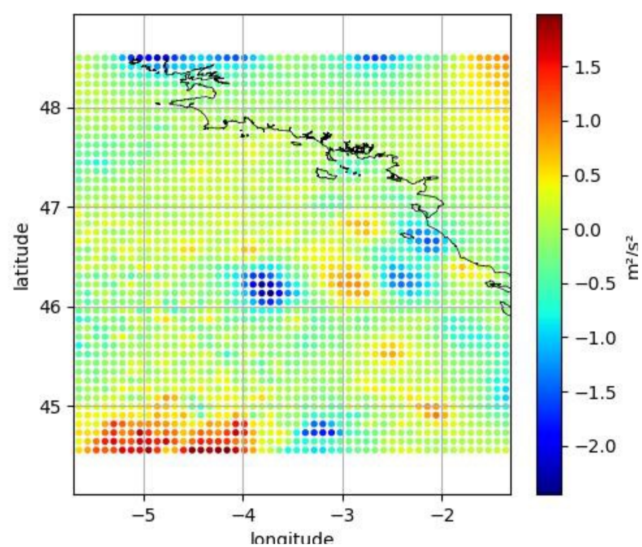
only from data δg



Bias: -0.9 m^2/s

RMS: 1.3 m^2/s^2

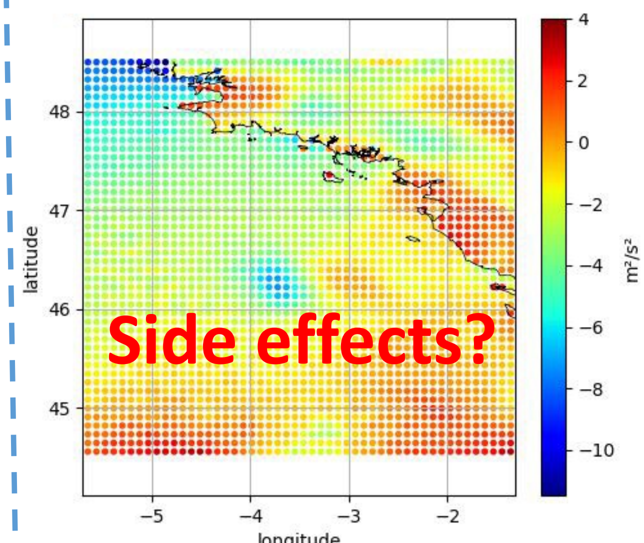
from data δg+T



Bias: -0.06 m^2/s^2

RMS: 0.5 m^2/s^2

from data δg



Bias: -1.1 m^2/s^2

RMS: 1.6 m^2/s^2

How to choose location of the clock points to improve the reconstruction of T ?

Principle of Genetic Algorithms (GA)

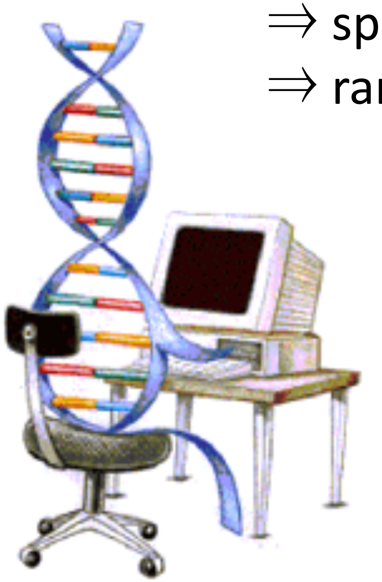
- Based on Darwin's theory of natural selection and modern genetics:

→ Natural selection: best adapted individuals to their environment ("fit") survive longer and reproduce more easily

→ Genetic: the laws of heredity use notions of crossover-mutation. The evolution and the adaptation of the species is explained by referring the lucky factor :

⇒ species evolve without a predetermined goal

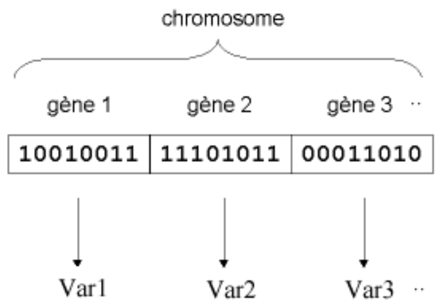
⇒ random spontaneous births



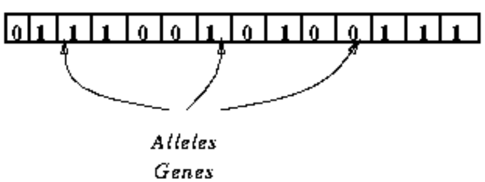
- Idea: solving complex optimization problems by simulating the process of biological evolution

Elements of a Genetic Algorithms (GA)

COMPUTER

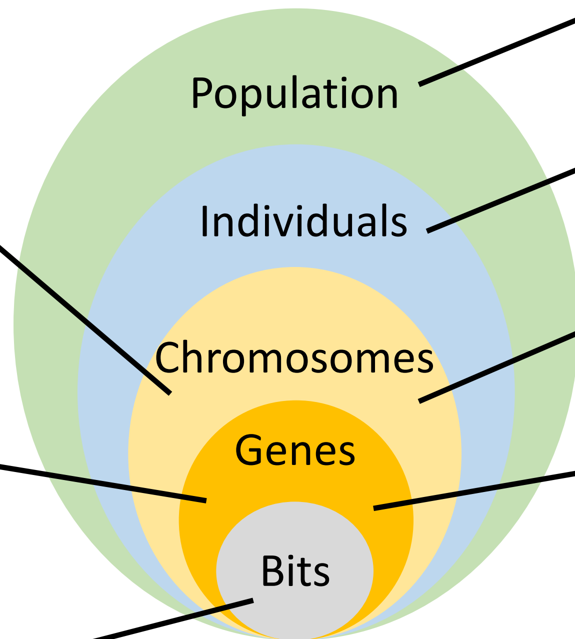


Position of 1s and 0s control the characteristic of the problem



Binary encoding (0,1)

NATURE



Set of solutions

Solution to the problem

Encoding for a solution

Part of the encoding solution

- Encoding depends entirely on the problem heavily !
- Binary is the most common method

In practice

List of location points and data:

LAT (deg)	LON (deg)	H (m)	T (m^2/s^2)	Noise T
45.621496	4.040226	535.515300	-0.145871	0.100000
46.111996	2.898492	524.439286	0.162066	0.100000
...				
44.737321	1.439303	328.054267	1.241396	0.100000

ID	Binary string
53	00110101
149	10010101
...	
581	11001001

- Each data point is recognized by an integer ID (line number for example)
- The ID of the gene is encoded in binary string
- Decoding function d for a gene of length l is:

$$x = d(A) = \sum_{i=1}^{l} a_i 2^{l-i-1}$$

Ex: $A=\{a_i\}=\{1,0,1,1\}$ → x=11

- Accuracy of the decoding s (depends on the number of points to be tested):

$$X' = x_{min} + x \frac{x_{max} - x_{min}}{2^s - 1}$$

Ex: if we have 1056 potential locations

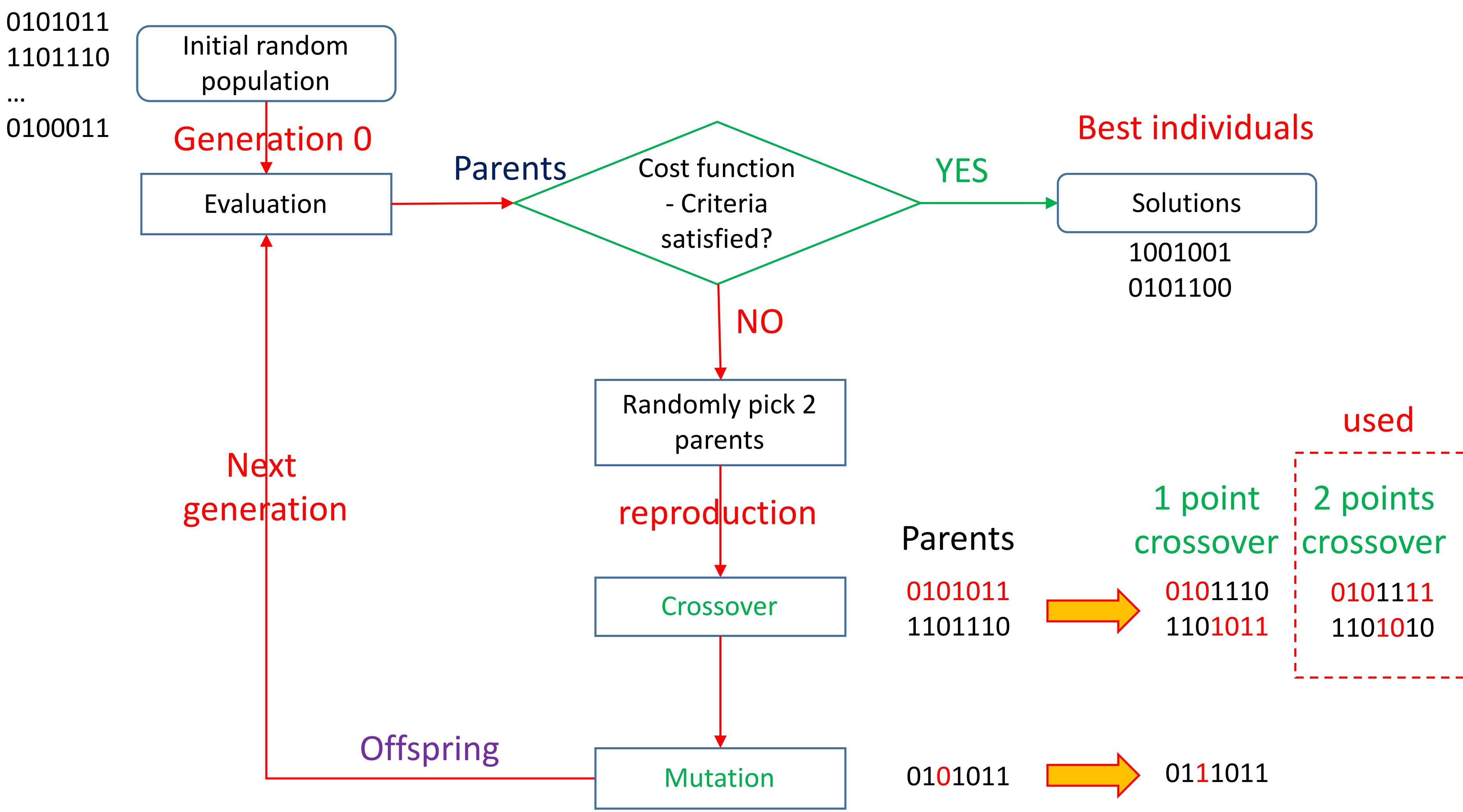
→ ID = [x_{min} = 0; x_{max} = 1055]

→ Encoding of the IDs with s=11 ⇒ 2^{11} = 2048 bits

→ The GA pulls the gene A = {1111010} → x = 122 → X '= 62.8 ⇒ 62

→ Accuracy of 1/2048 in the search space

Classical scheme associated with GAs



Pareto dominance

- Context

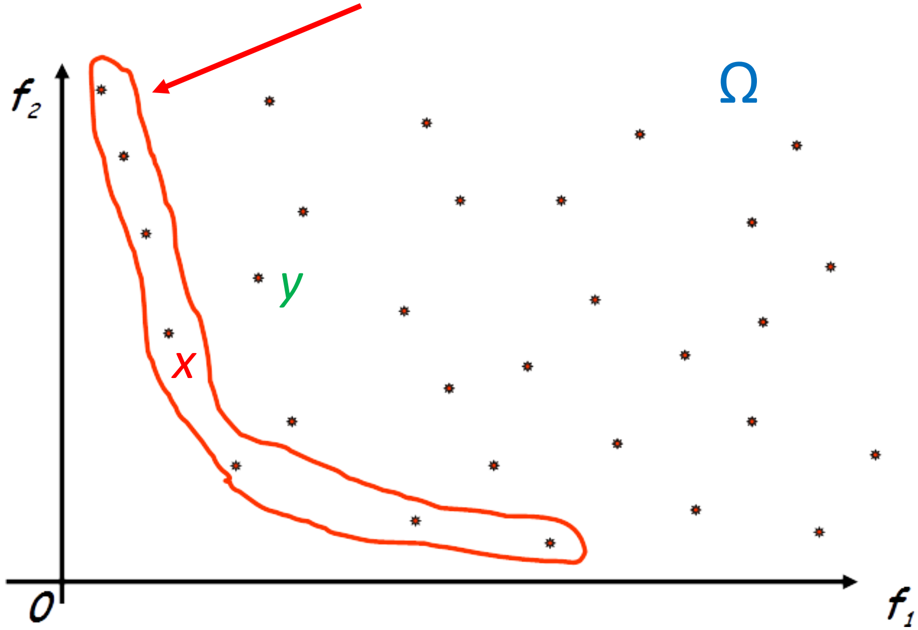
Minimization of p objective functions f_i defined on $\Omega \in R^n$

- Dominance in the sense of Pareto

Partial order relationship defined by $x < y \; ssi \begin{cases} \forall i \in [1,p] \, f_i(x) \leqslant f_i(y) \\ \exists j \in [1,p] \, f_j(x) < f_j(y) \end{cases}$

- Pareto front-set

Set of non-dominated solutions of Ω
Image in the search space = Pareto front



y dominated by x

ε-MOEA

- ε-MOEA : ε Multi-Objective Evolutionary Algorithm
- Developped by **Pr. Deb (2003)**
- Recent genetic algorithm, faster and with very good convergence compared to other schemes (McMc, NSGA-II, etc.)
- **Characterized by 2 main principles**: ε-dominance and the archive

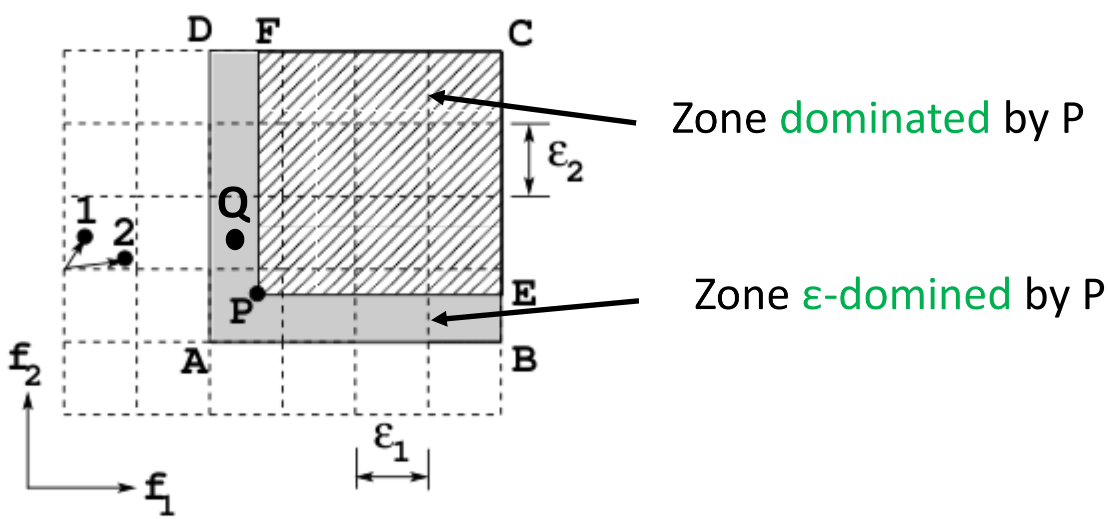
ε-dominance [Laumanns et al., 2003]

- Modified dominance relation (ε_i, real p fixed)

$$x \prec y \; ssi \begin{cases} \forall i \in [1,p] \, f_i(x) \leqslant f_i(y) + \epsilon_i \\ \exists j \in [1,p] \, f_j(x) < f_j(y) + \epsilon_j \end{cases}$$

- Ex: for 2 objectives

ε = tolerance on the value of the objectives (resolution):
- allows diversity and influences the size of the Pareto front
- eliminates points that are too close together



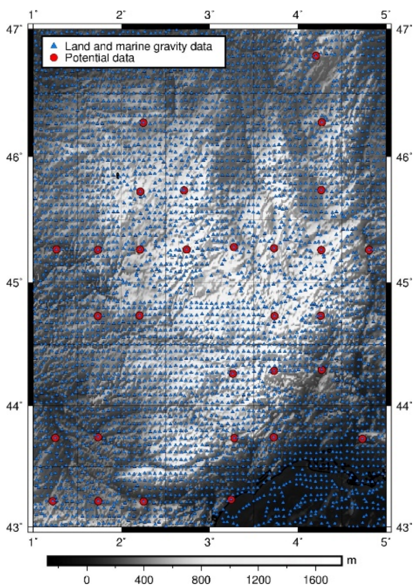
Although solutions P and Q are non-dominated with each other,
P ε-dominates Q since the latter belongs to the region ε-dominated by P

BINARY CODING + chromosomes with fixed lengths

Optimization of the position of 33 clocks

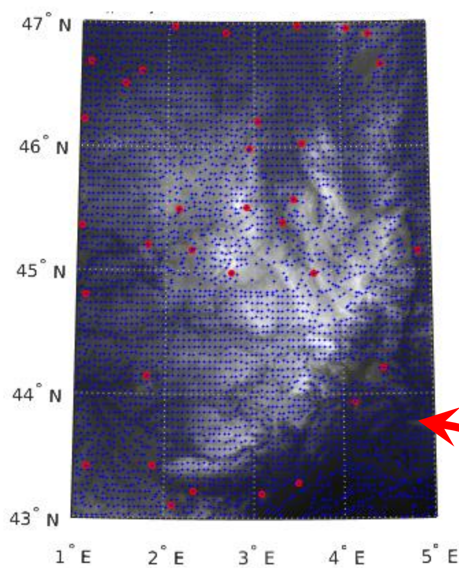
- Cost function (objectives)
  - → Minimization of the reconstruction residuals on T (bias $|\mu|$ and STD σ)
- Constrains
  - → Clock point at the same place as a gravity point
  - → Clock point in an area poorly covered by gravity
  - → Minimum distance between 2 clock points
  - → Clock points on ground

μ=2.40E-3 - σ=7.3E-2



“Handmade”

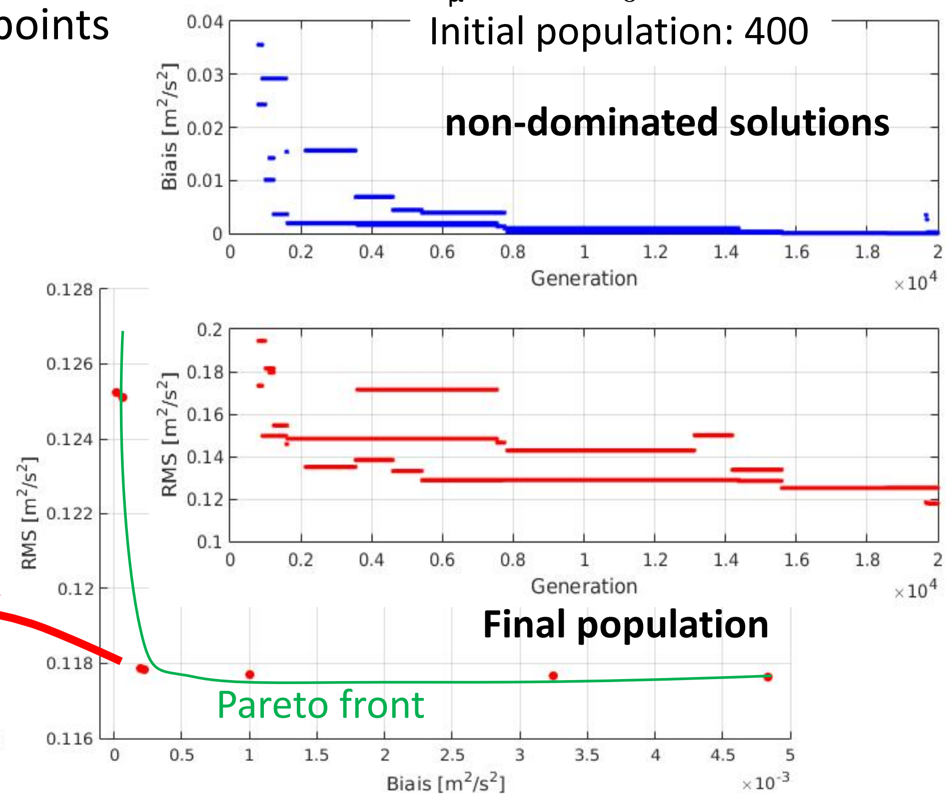
μ=1.2E-4 - σ=8.2E-2



“Genetically” modified

ε_μ=1.E-6 ; ε_σ=1.E-2

Initial population: 400



Optimization of the clocks point location by genetic algorithms

Integer encoding with variable length chromosomes

- Need to customize the GA to suit the problem or class of problems under consideration
- Integer encoding → to increase the efficiency of the genetic algorithm, i.e. better convergence: each clock point is associated with its position in the sequence
- Variable length strings → to minimize the number of observables

- Set a limit on maximum length of chromosome (e.g. 16 location points)

- Two-point crossover

String completed by 0s

Parent A	1	5	6	8	10	15	16	23	26	0	0	0	0	0	0	0
Parent B	3	7	8	9	11	13	14	19	21	22	23	27	31	0	0	0
Offspring	1	5	6	8	10	13	14	19	21	22	0	0	0	0	0	0
	3	7	8	9	11	15	16	23	26	0	23	27	31	0	0	0

Genes are then sorted in ascending order

- Mutation

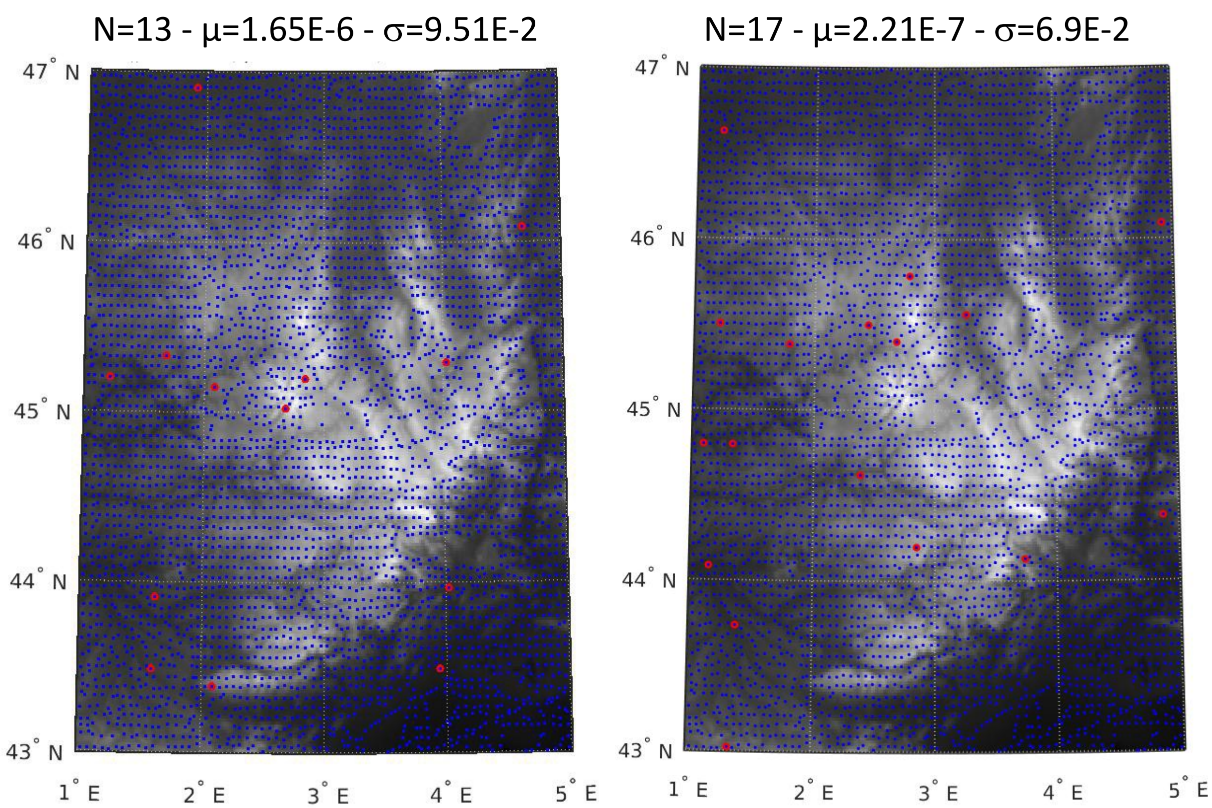
3	7	8	9	11	15	16	23	26	27	31	0	0	0	0	0
3	7	28	9	11	15	16	23	0	27	31	0	0	1	0	0

change annihilation creation

INTEGER CODING + chromosomes with variable lengths [Coulot et al. 2015]

- Cost function (objectives)

→ Minimization of the reconstruction residuals of T (bias $|\mu|$ and STD σ)
→ Minimization of the number of clock point location N = [5; 50]



ε_μ=1.E-6 ; ε_σ=1.E-2 ; ε_N =0.5
Initial population: 400
Number of generation : 20000
CPU time: about 8 days

→ Small number of clock points
→ Reduction of the bias by 4 orders of magnitude than previous scenarios

Better coverage

(article in process…)

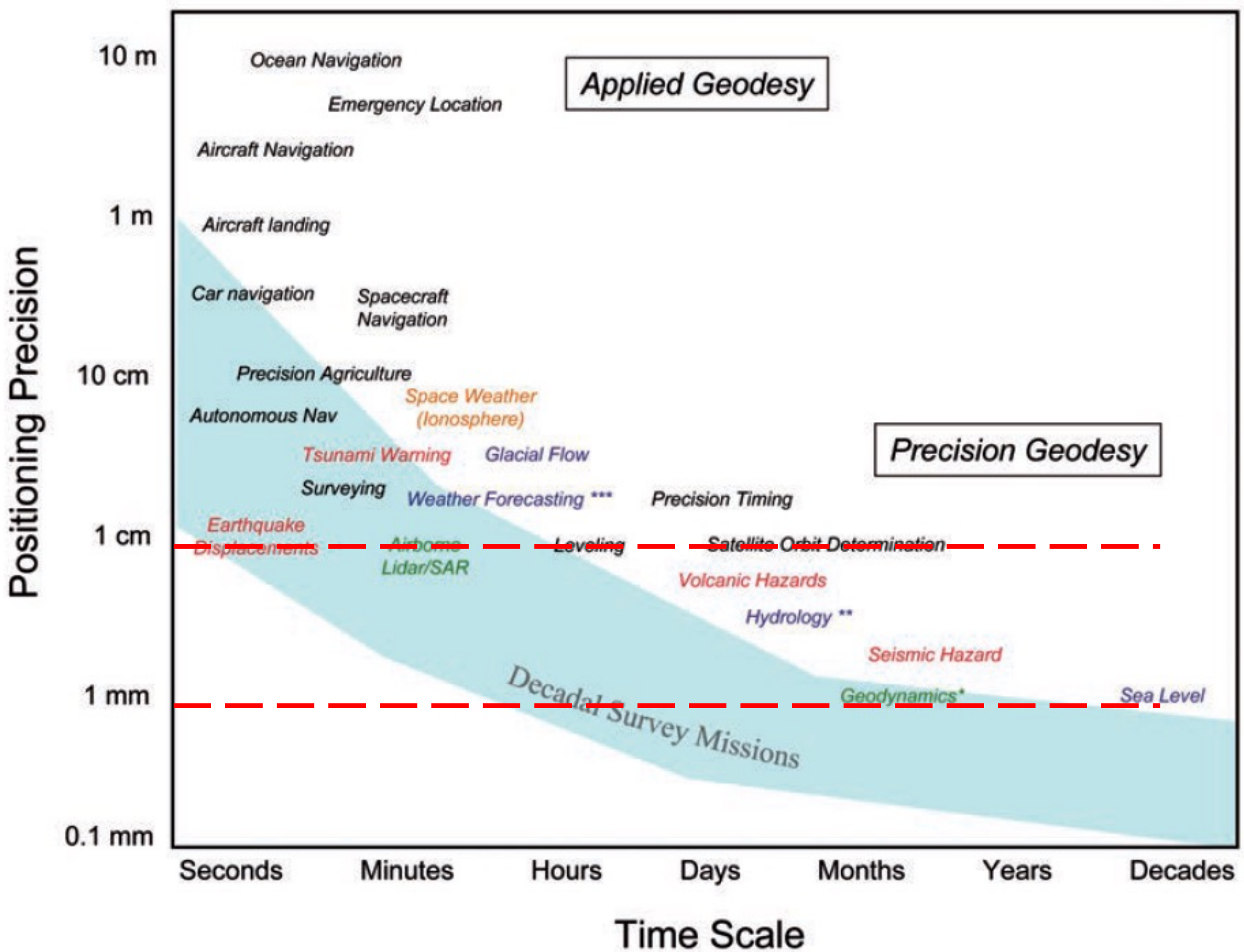
Conclusions

- ✓ Clocks provide complementary information to surface and satellite data, particularly in areas poorly covered by gravity data
- ✓ In our study area, around 30 clock points improve the potential reconstruction (by more than 2 orders of magnitude) and improve accuracy bias (by a factor 3)
- ✓ Results also confirmed in Alps-Mediterranean region, Bretagne and Atlantlic coast
- ✓ A GA is a powerful tool to find optimal data coverages (of any kind): smaller residuals on T with 50% less clock points wrt the first results

Future prospects

- Cost function of the GA evaluated locally in sub-areas (in progress…)
- Use of the formal errors of the LSC method instead of a computed reference solution (in progress…)
- Spatially correlated error patterns (colored noise) to identify biases in dataset
- Test different a priori models (isotrop and non-isotrop)
- Optical clocks on the littoral zone ? On board ship ?
- Sensitivity of clocks to the vertical displacement
- Can optical clocks (10^{-19} – 10^{-20}) could detect geodynamic processes inducing Earth deformations ? (mass transport, seismic cycle, surface loading, hydrological cycle, …)
- **Other ideas ?**

National Report Council (NRC) on Precise Geodetic Infrastructures, 2010

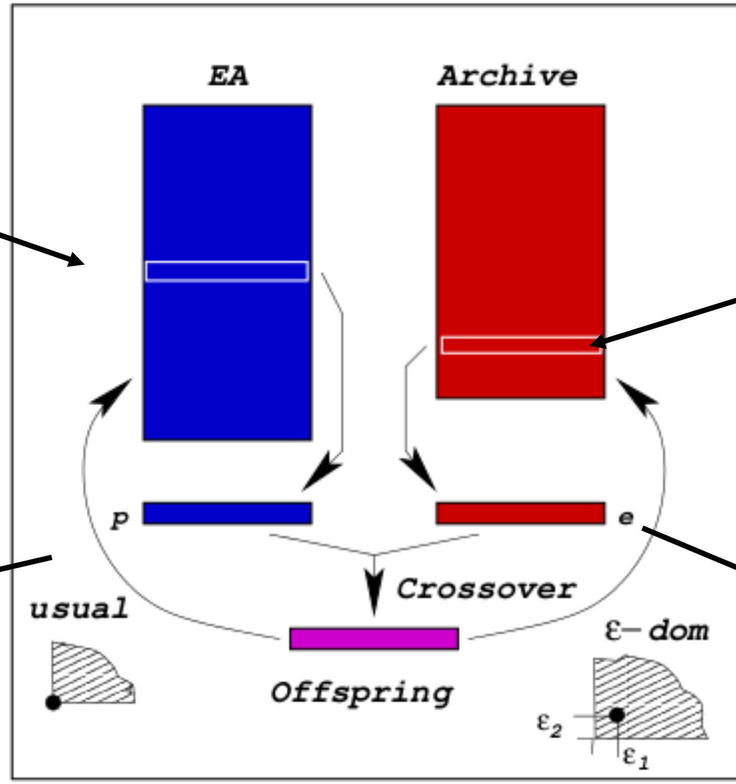


Optimization of the clocks point location by genetic algorithms

a: archive
o: offspring
p: parent

ε-MOEA

2 random individuals
Tournament:
- one dominates the other
- otherwise, chance



1 random individual

Comparison dominance

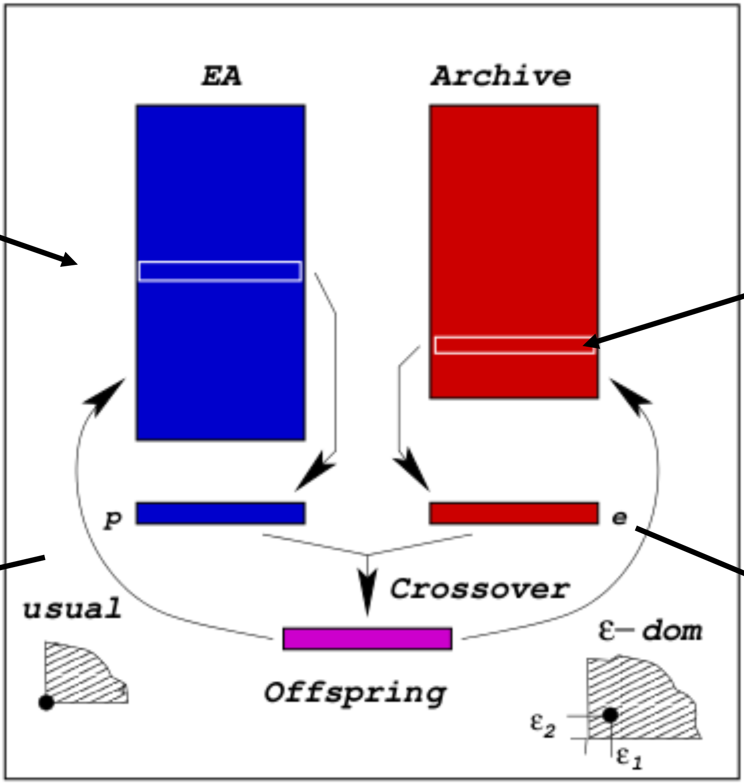
Comparison ε-dominance

Comparison of o with all p's of P (population size kept constant)
- o dominates one of the p ➔ o replaces p
- o is dominated by a p ➔ o is not retained
- Otherwise, o replace a randomly chosen p

a: archive
o: offspring
p: parent

ε-MOEA

2 random individuals
Tournament:
- one dominates the other
- otherwise, chance



1 random individual

Comparison dominance

Comparison ε-dominance

Comparison with the archive (variable size controlled by the value of ε) of o with all of A's
- o ε-dominates one of the a ➔ o replaces a
- o is ε-dominated by a ➔ o is not retained
- Otherwise, 2 cases can occur:
  - -> o is in the same box as a of A : we compare o and a with dominance
  - -> None of A shares box with o ➔ o is accepted

Contribution of atomic clocks for geopotential determination

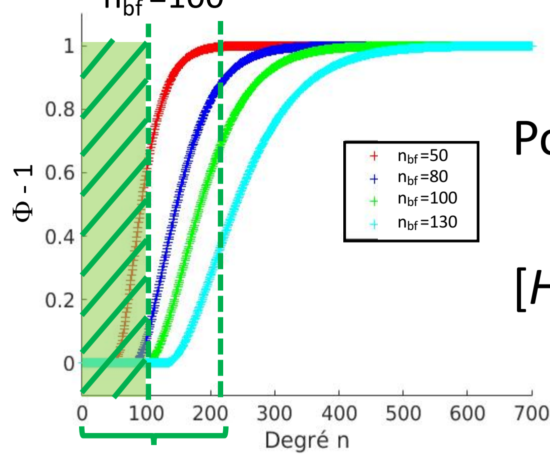
Step 1

Generation of the anomalies δg and T at the Earth's surface (ellipsoidal approximation)

- Global gravity model at 10 km resolution (n=2000)
  → EIGEN-6C4 [*Förste et al., 2014*]

- Removal of low frequencies

n_{bf} =100



Satellites coverage

Poisson wavelet spectrum:

$$\Phi(t) \propto t^3 \exp(-t)$$

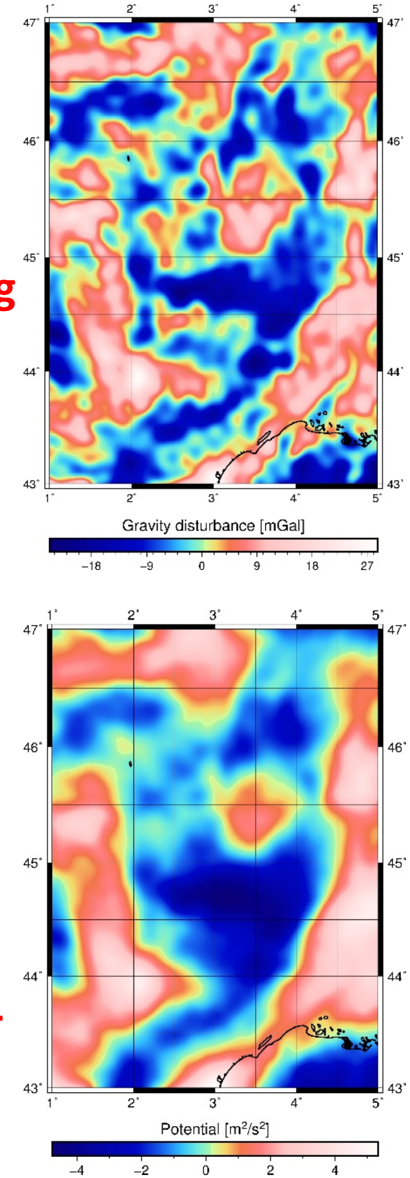
[*Holschneider et al., 2003*]

- Subtract the terrain effects
  → dV_ELL_RET2012 [*Claessens and Hirt, 2013*]

Synthetic field (reference model)

δg

T



Assumption on the gravity field a priori regularity

→ Logarithmic 3D covariance function [*Forsberg, 1987*]

$$C'(x,y,z_1+z_2) = f\sum_{i=0}^{3}\alpha_i C(x,y,z_i)$$

with

$$x = x_2 - x_1; \qquad y = y_1 - y_1$$
$$z_i = z_1 + z_2 + D_i$$
$$\alpha_i = \{1,-3,3,1\}$$
$$D_i = D + iT$$
$$f = C_0 \log^{-1}\left(\mathrm{D}_1^3 D_3 \middle/ D_0 D_2^3 \right)$$

- 3 parameters to be estimated:

- C_0: variance of δg
- D and T: shallow and compensate depth parameter

- Auto-covariance function (ACF) for δg:

$$C = C_{zz} = \frac{\partial}{\partial z_1}\frac{\partial C_{TT}}{\partial z_2} = -\log(\mathrm{D}_i + r_i)$$

with

$$r_i = \sqrt{d^2 + D_i^2};\ d^2 = x^2 + y^2$$

- ACF for T:

$$C = C_{TT} = \iint C_{zz} dz_1 dz_2 = \frac{3}{4} z_i r_i + \left(\frac{r_i^2}{4} - \frac{3}{4} z_i^2 \right) \log(\mathrm{z}_i + \mathrm{r}_i)$$

- Auto-correlation function between T and δg:

$$C = C_{TT_z} = r_i - z\log(\mathrm{z}_i + \mathrm{r}_i)$$

Step 3

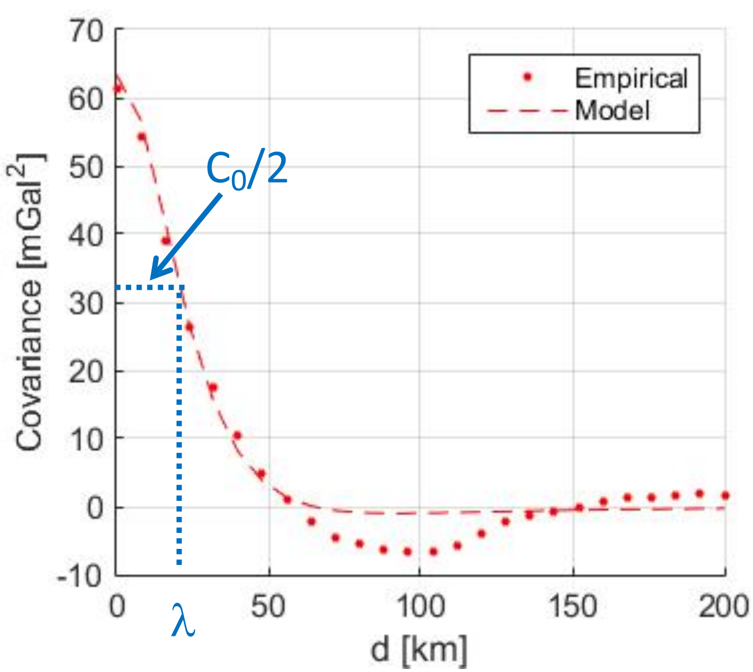
- Assumption on the gravity field a priori regularity

→ Logarithmic 3D covariance function [*Forsberg, 1987*]

- C_0: variance of δg
- *D* and *T*: shallow and compensate depth parameter

3 parameters to be estimated

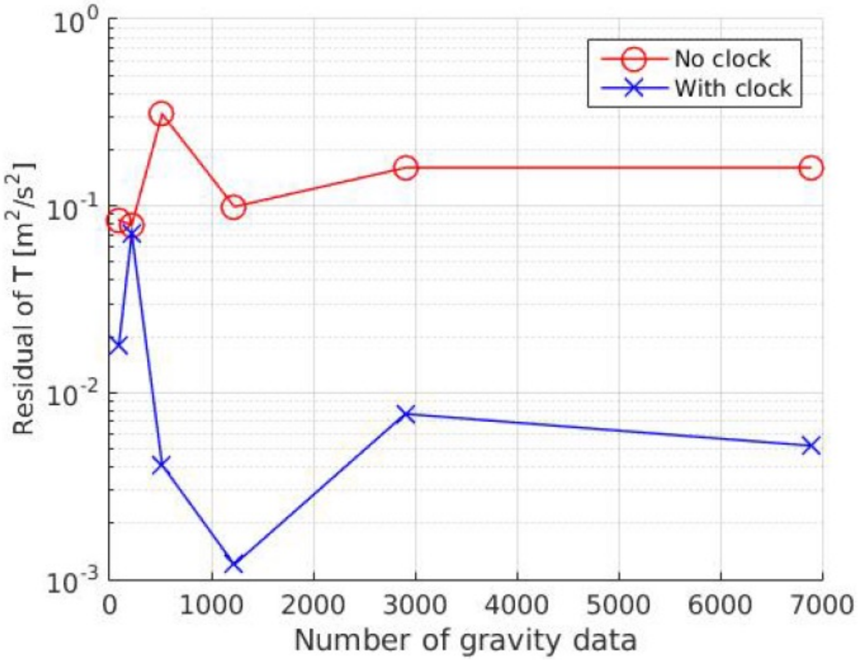
Parameters estimation of the covariance function from the empirical ACF δg



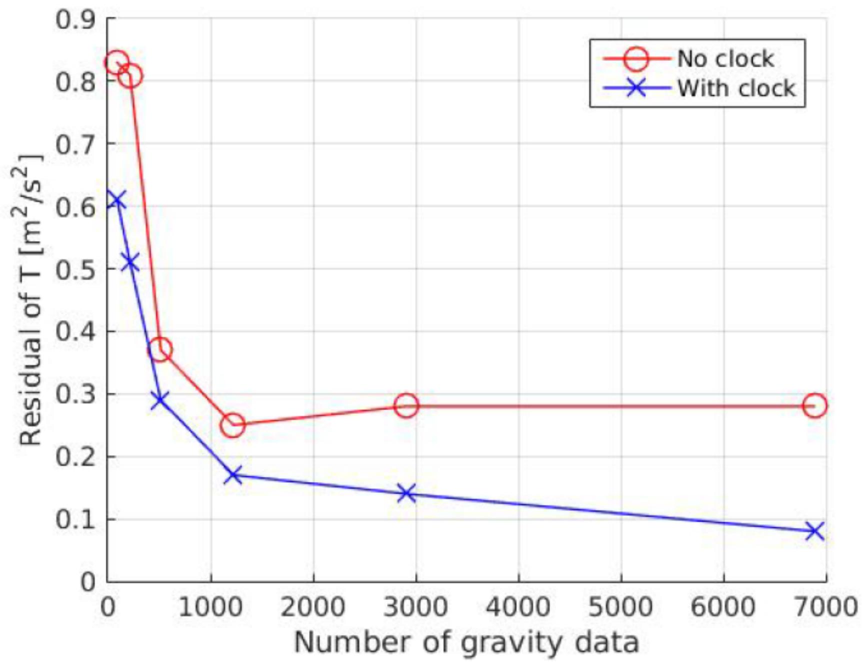
- C_0 = 63.35 $mGal^2$ (estimated)
- D = 24 km
- T = 15 km

Correlation length: λ≈21km

Effect of the number of gravity data



(a) Absolute value of the mean.



(b) Rms.

→ Adding 38 clocks measurements reduces significantly the trend from the modeling error

→ Increasing the gravity data resolution has more impact on the interpolation errors at medium to smaller scales

→ Adding clocks leads to a further reduction of the rms of residuals