Using optical clocks and quantum gradiometers onboard satellites for determining the Earth's gravity field

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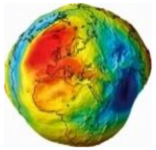
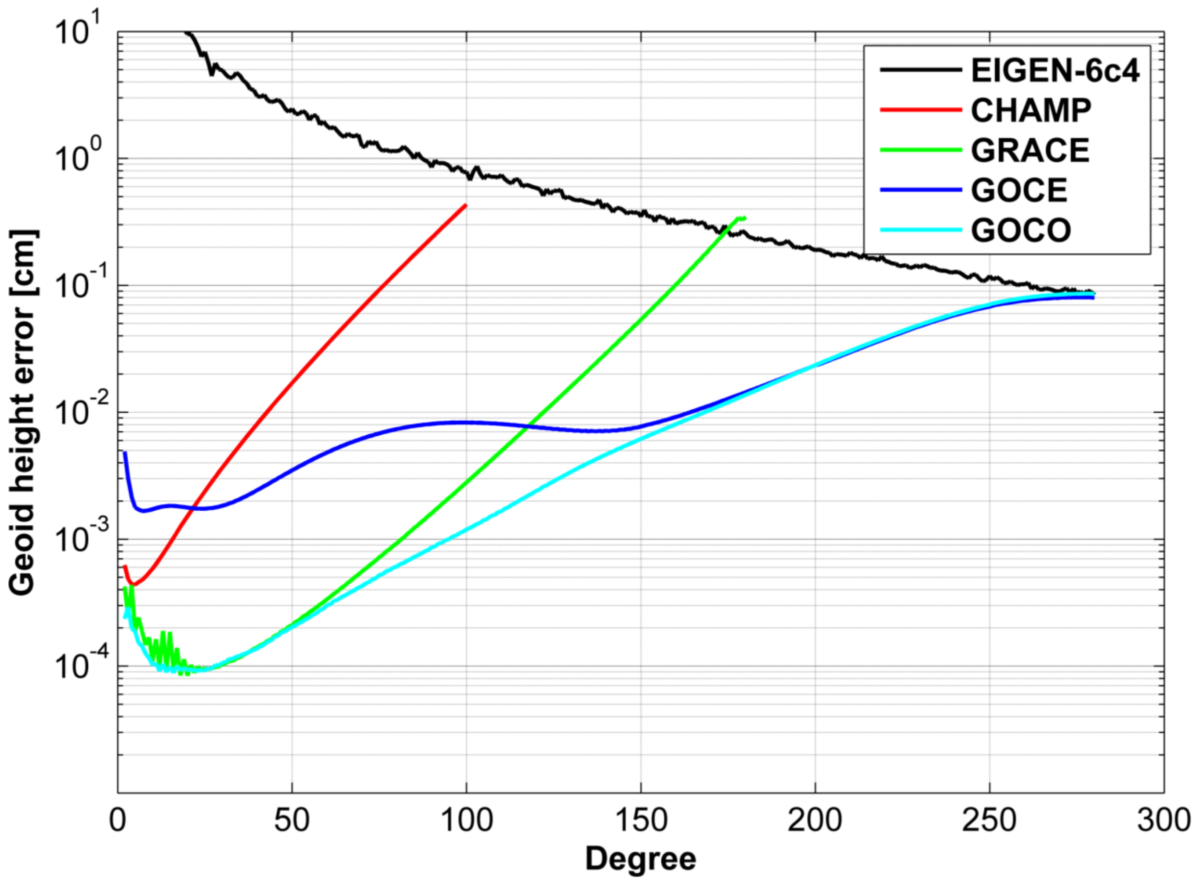


Outline

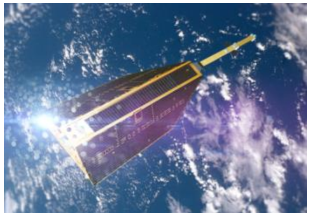
1. Motivation
2. Gravity field modelling
   - Optical clocks
   - Cold Atom Interferometry (CAI) gradiometry
   - Combined analysis
3. Conclusions and future perspectives

Motivation

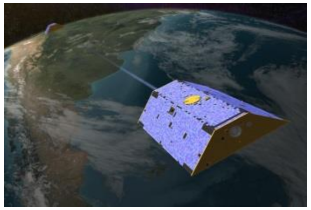
Development of Earth gravity field models



CHAMP (2000 – 2010)



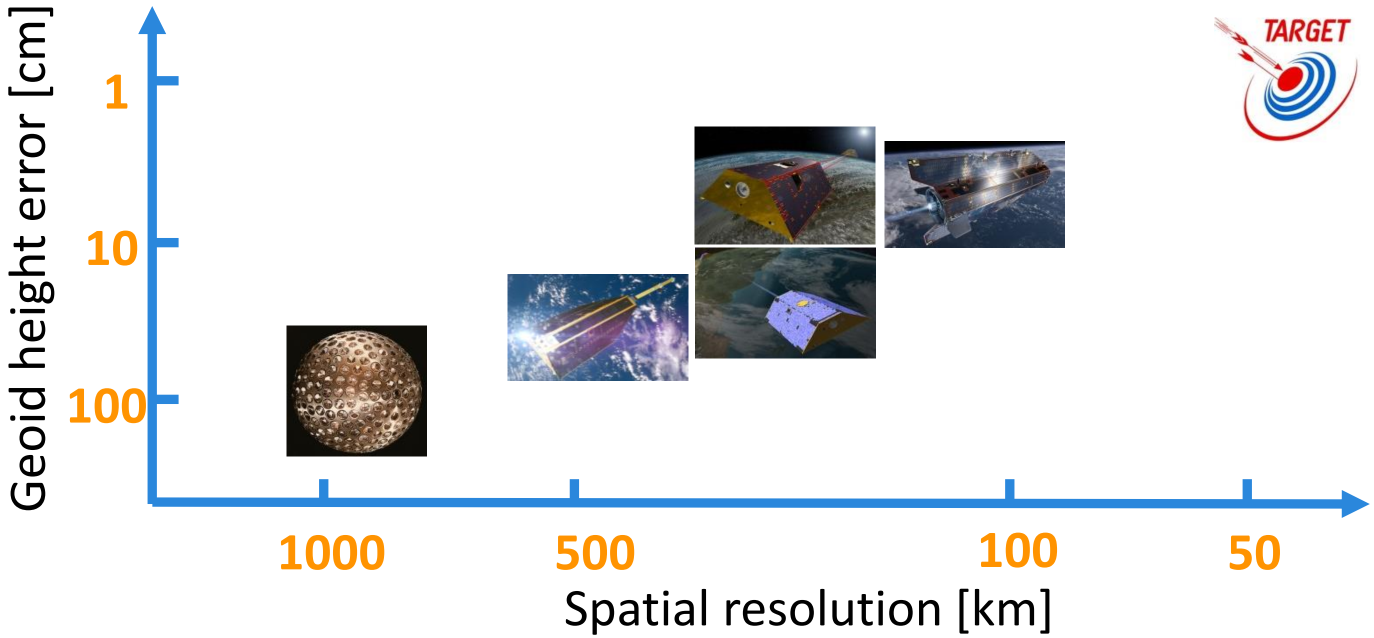
GRACE (2002 – 2017)



GOCE (2009 – 2013)

Motivation

Future goals: higher accuracy and better spatial-temporal resolution!

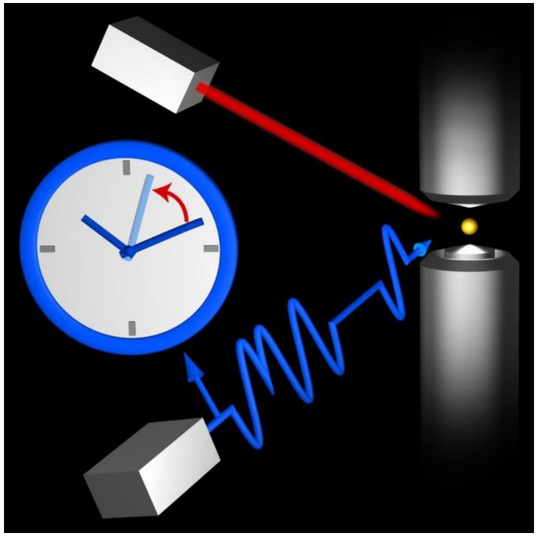


New measurement concepts or improved instrumentation for future satellite gravity missions.

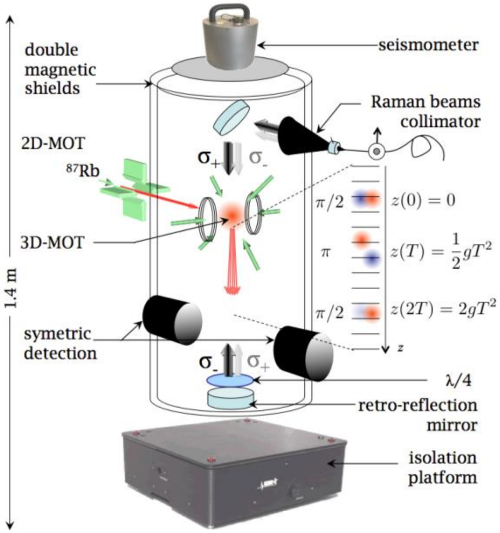
Motivation

Quantum sensors

Optical clocks



Atom interferometry gravimeters



Great potential of quantum instruments and methods for the determination of the Earth gravity field.

The global gravity field is expressed as:

$$T = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)]\,\bar{P}_{nm}(cos\theta)$$

Retrieve the Earth gravity field by observing:

- potential values (T);
- accelerations ($T_i = \frac{\partial T}{\partial r_i}$);
- gradients ($T_{ij} = \frac{\partial^2 T}{\partial r_i\,\partial r_j}$);

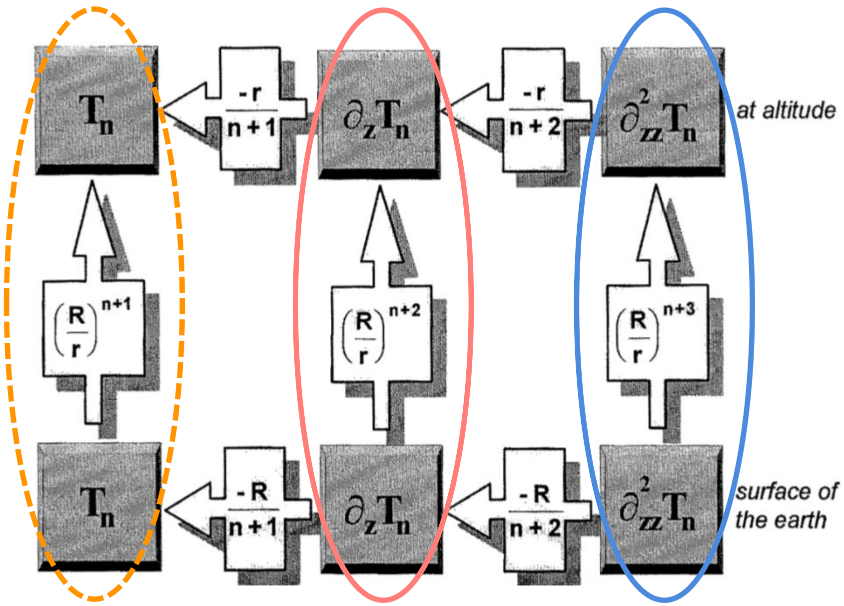


Figure is taken from R. Rummel (1997).

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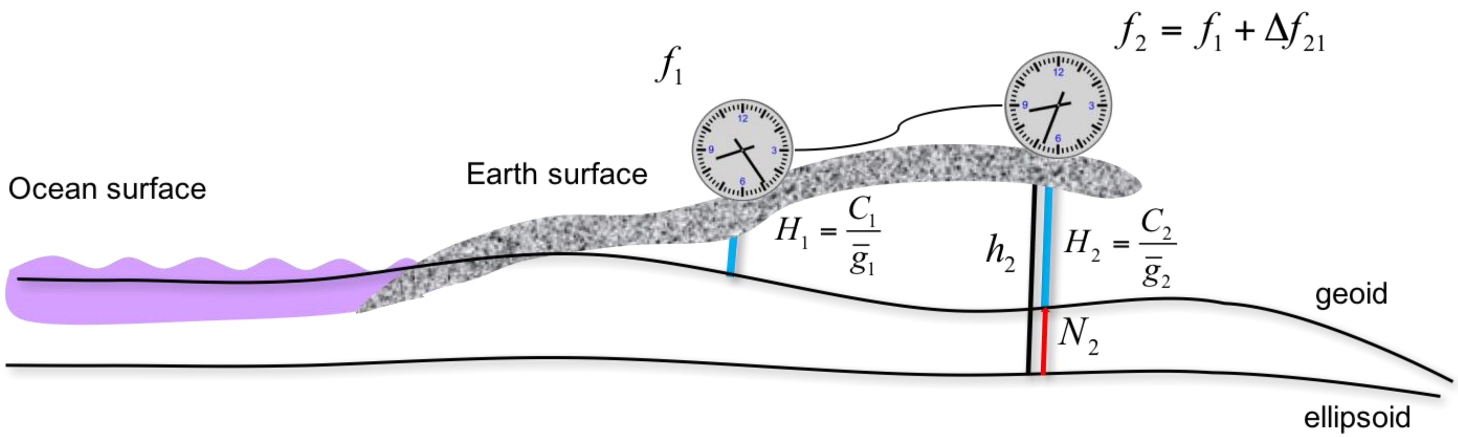
Relativistic geodesy

Basis: Einstein's General Relativity Theory

Gravitational redshift: $\frac{\Delta f_{21}}{f_1} = \frac{f_2 - f_1}{f_1} = \frac{W_2 - W_1}{c^2} + O(c^{-4})$

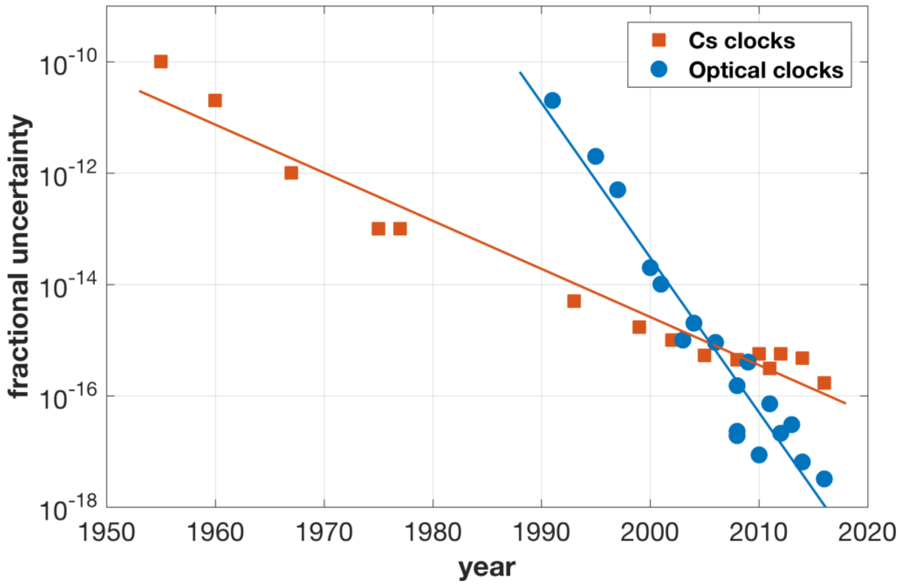
Error propagation: $\frac{\Delta f}{f}\ (1.0 \times 10^{-18}) \sim \Delta W\ (0.1\ \mathrm{m}^2/\mathrm{s}^2) \sim \Delta h\ (1.0\ \mathrm{cm})$



Differences of the gravity potential can **directly be obtained** by the comparison of frequencies!

Instrumentation

Clock performance



Link techniques

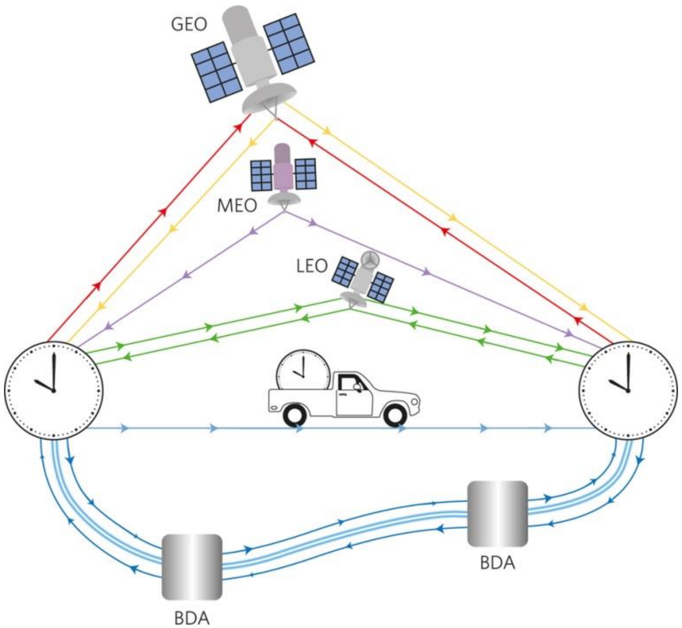


Figure is taken from F. Riehle (2017).

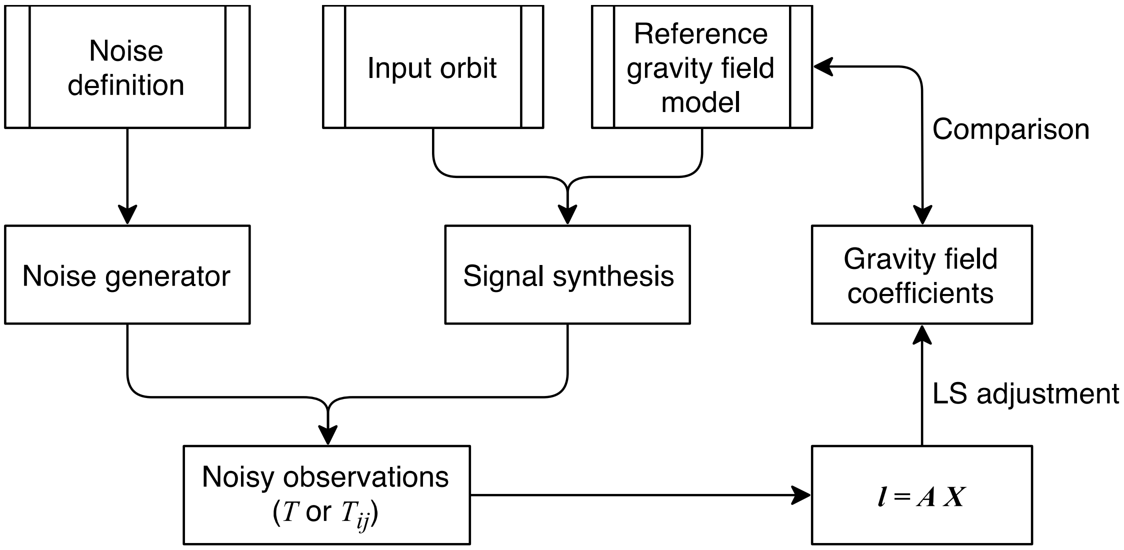
Clocks and various frequency links are approaching to provide frequency comparisons **at the level of 10^{-18}**.

Simulation

We ran simulations to explore the potential of clocks in space for gravity field recovery, with input:

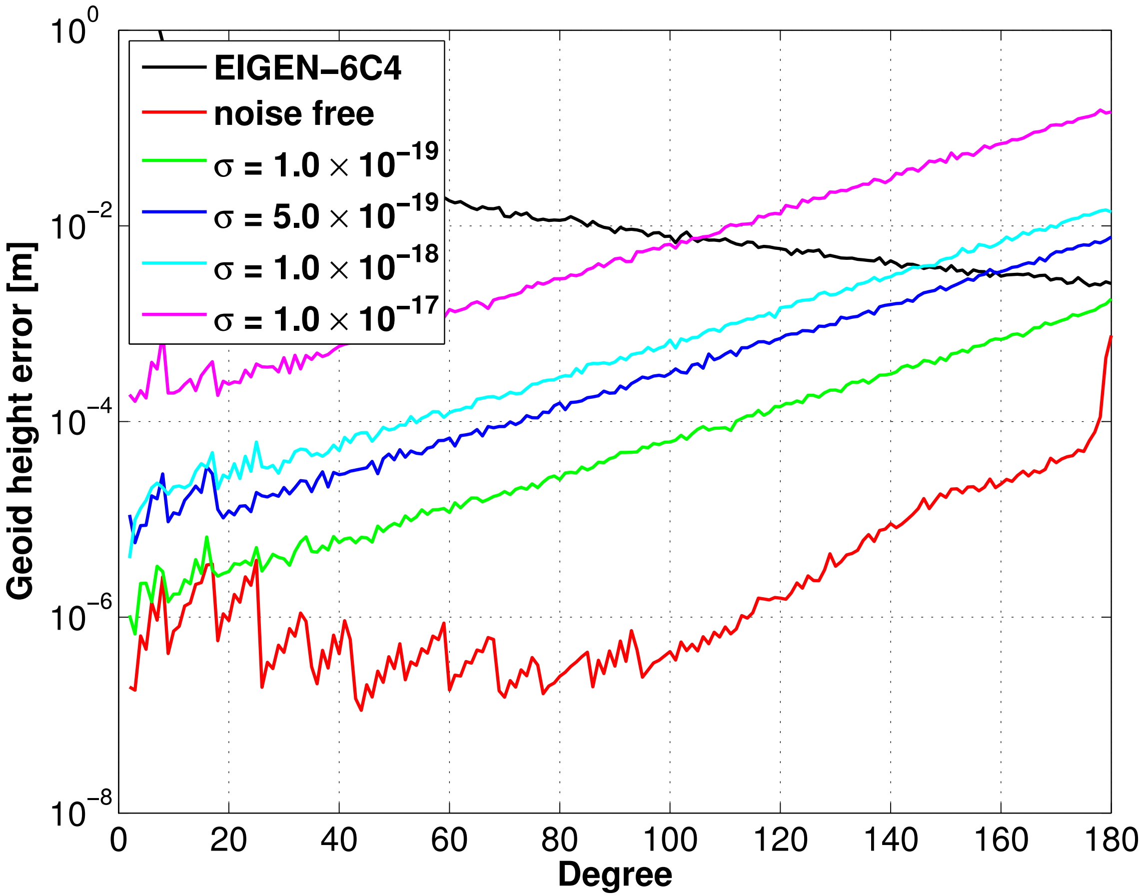
- Orbit: GOCE (Nov. and Dec., 2009), 5 s;
- Model: Eigen-6c4, d/o 360;
- Noise:
  - white;
  - colored.

The model was recovered to degree and order (d/o) **180.**



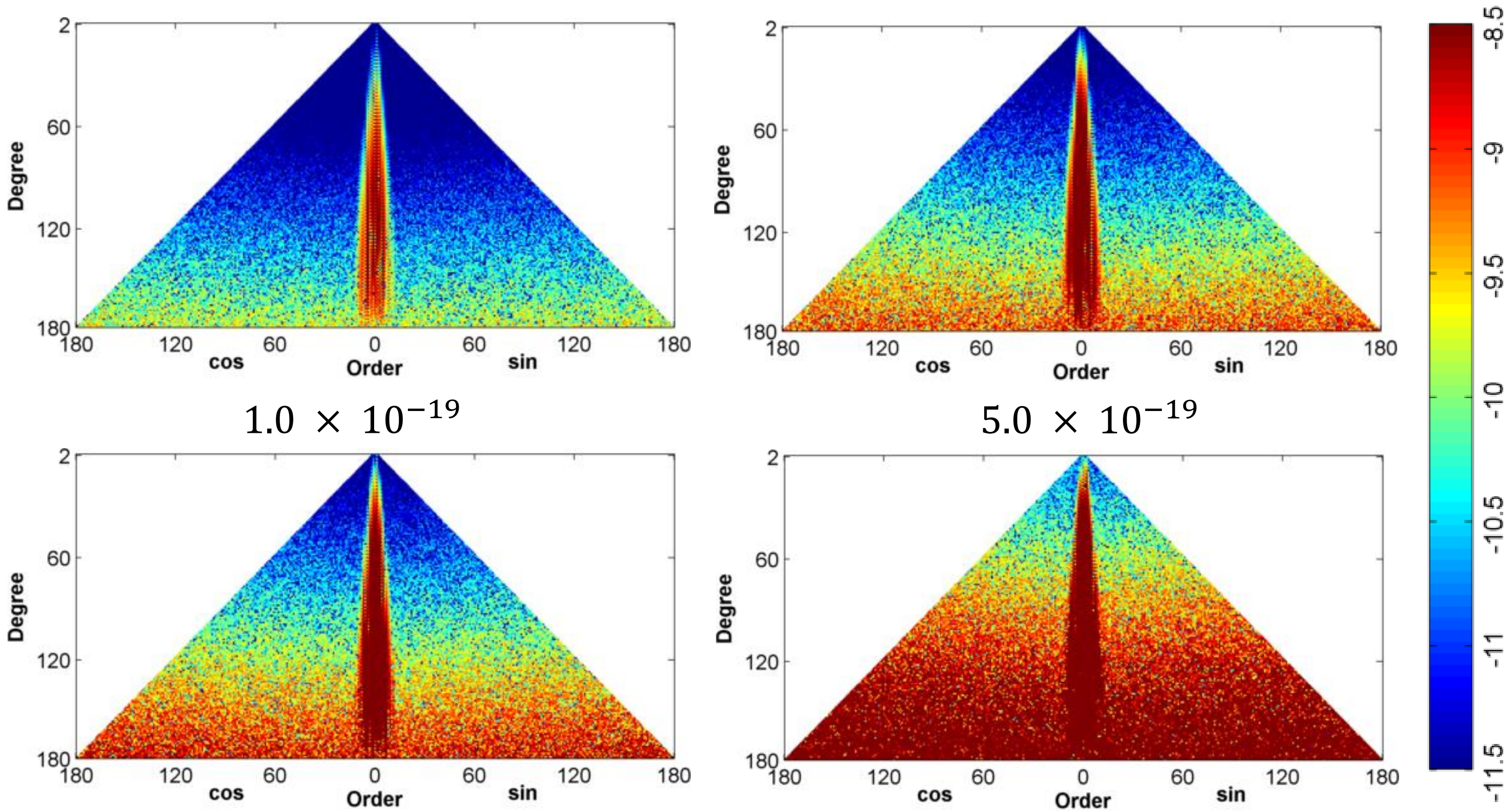
Results / white noise

Error degree variances for clock solutions



Results / white noise

Coefficient differences w.r.t. Eigen-6c4



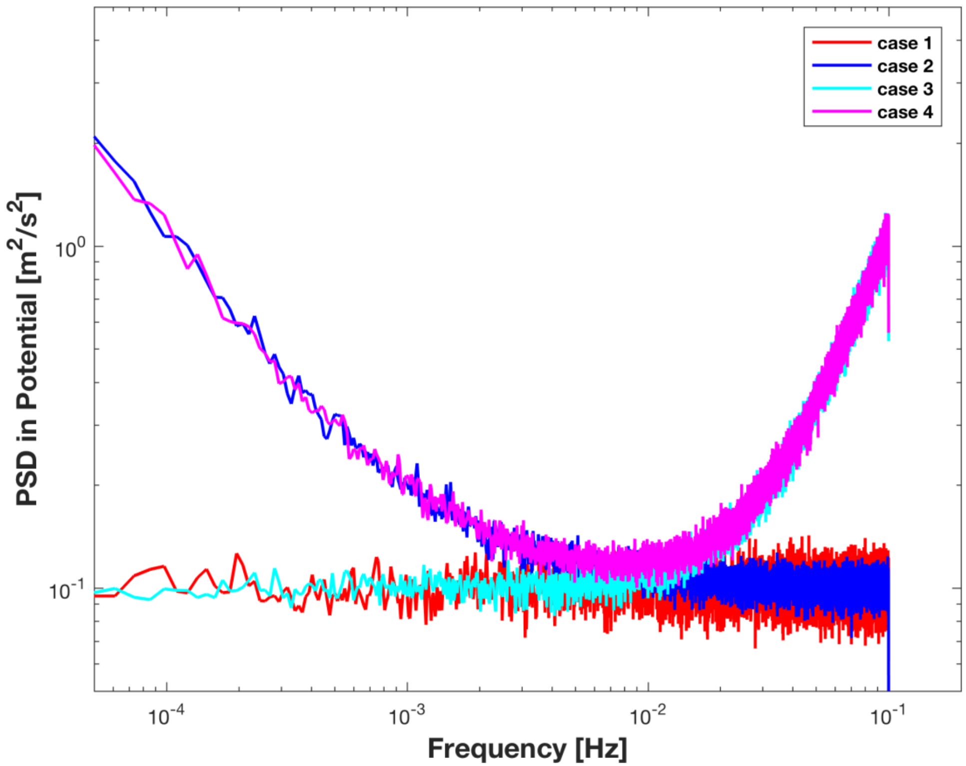
1.0×10^{-19} 5.0×10^{-19}

1.0×10^{-18} 1.0×10^{-17}

Results / colored noise

Colored noise was defined with the specified PSD function:

$$PSD = A \cdot (1.0 + k_1/f + k_2 f^2), \text{ with } A = 0.1\ \mathrm{m}^2/\mathrm{s}^2$$



- case 1: $k_1 = 0, k_2 = 0$
- case 2: $k_1 = 10^{-3}, k_2 = 0$
- case 3: $k_1 = 0, k_2 = 10^3$
- case 4: $k_1 = 10^{-3}, k_2 = 10^3$

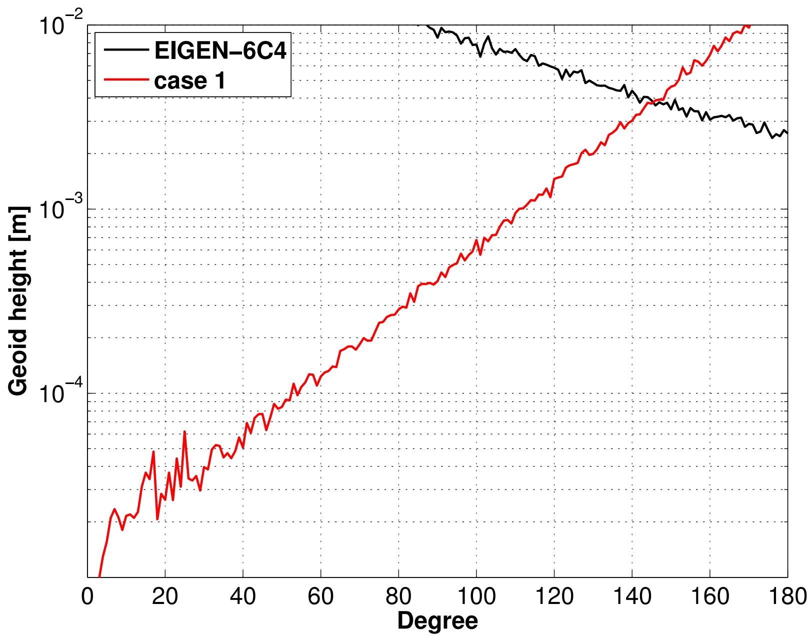
Results / colored noise

Colored noise was defined with the specified PSD function:

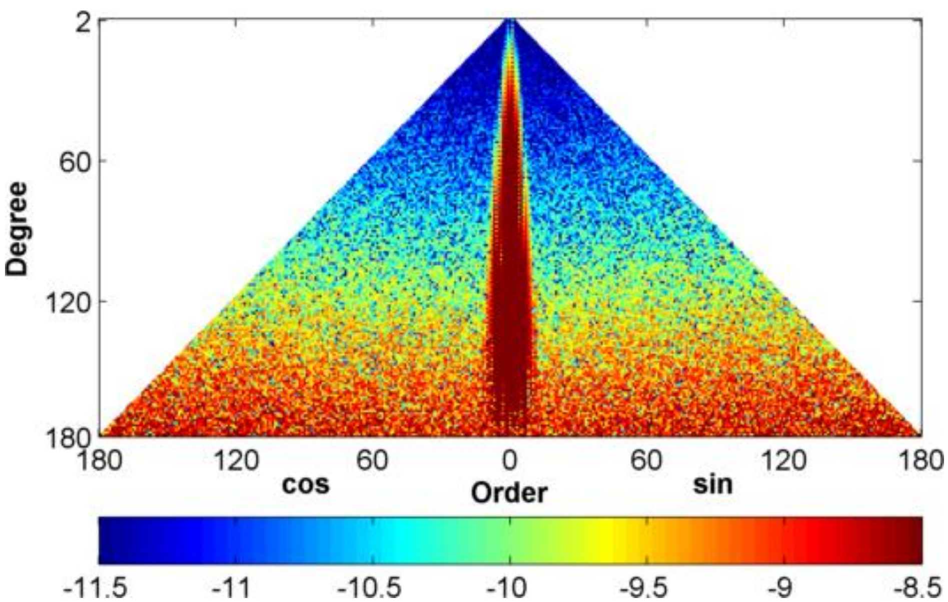
$$PSD = A \cdot (1.0 + k_1/f + k_2 f^2), \text{ with } A = 0.1\ \mathrm{m}^2/\mathrm{s}^2$$

Results for case 1: $k_1 = 0, k_2 = 0$

Degree error RMS



Coefficient difference



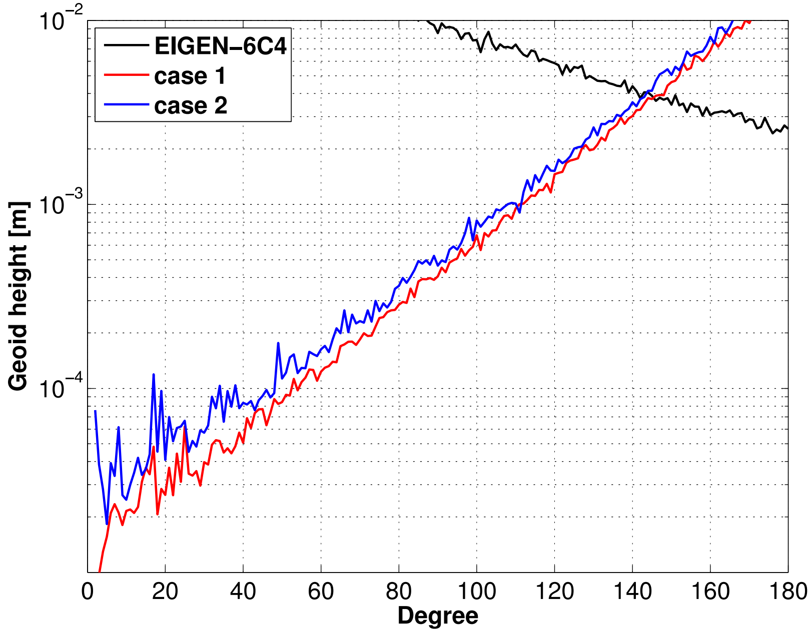
Results / colored noise

Colored noise was defined with the specified PSD function:

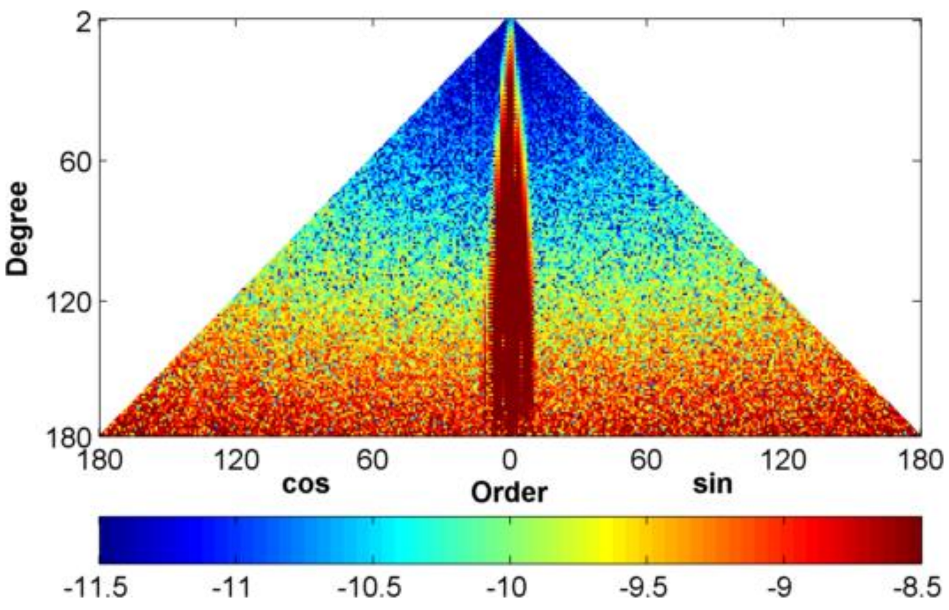
$$PSD = A \cdot (1.0 + k_1/f + k_2 f^2), \text{ with } A = 0.1\ \mathrm{m}^2/\mathrm{s}^2$$

Results for case 2: $k_1 = 10^{-3}, k_2 = 0$

Degree error RMS



Coefficient difference



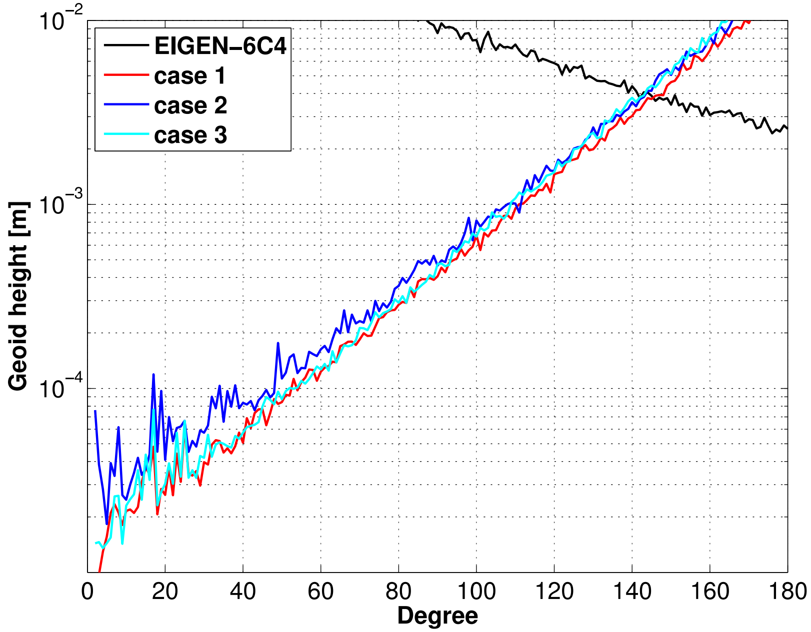
Results / colored noise

Colored noise was defined with the specified PSD function:

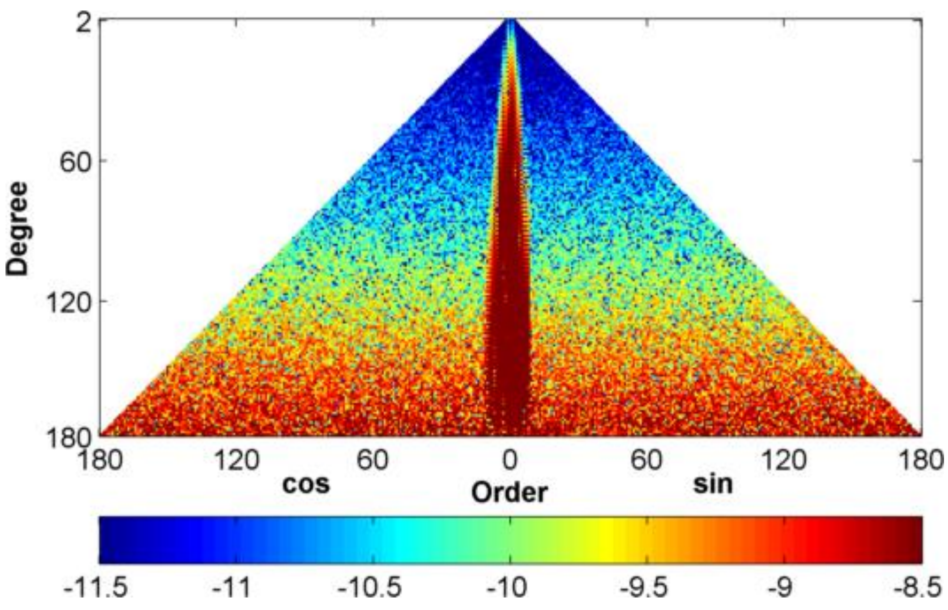
$$PSD = A \cdot (1.0 + k_1/f + k_2 f^2), \text{ with } A = 0.1\ \mathrm{m}^2/\mathrm{s}^2$$

Results for case 3: $k_1 = 0, k_2 = 10^3$

Degree error RMS



Coefficient difference



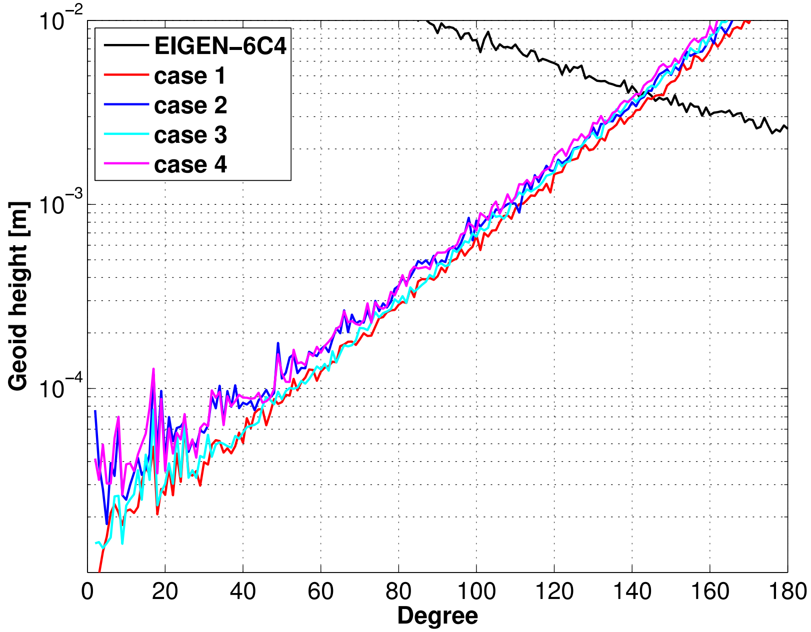
Results / colored noise

Colored noise was defined with the specified PSD function:

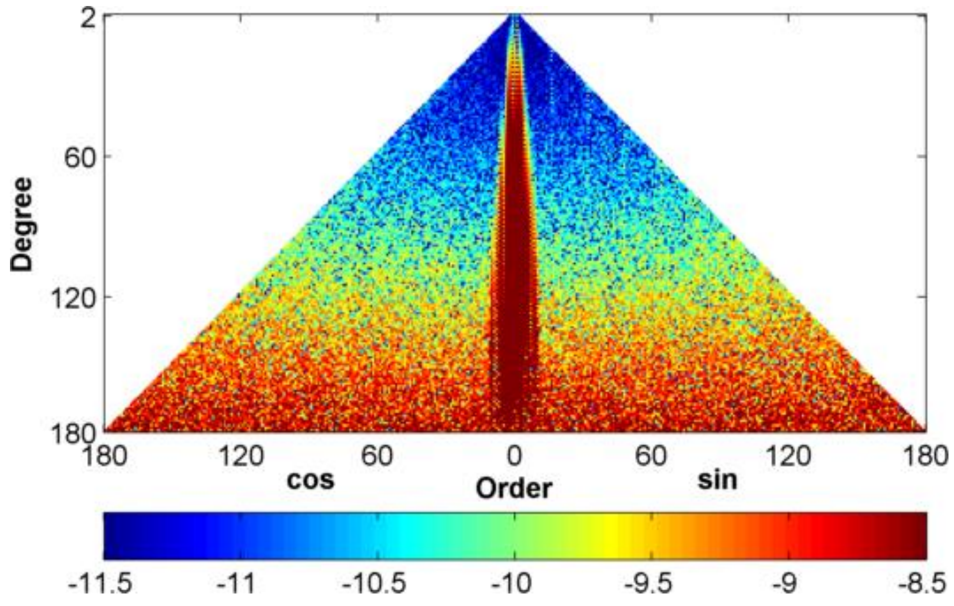
$$PSD = A \cdot (1.0 + k_1/f + k_2 f^2), \text{ with } A = 0.1\ \mathrm{m^2/s^2}$$

Results for case 4: $k_1 = 10^{-3}, k_2 = 10^3$

Degree error RMS



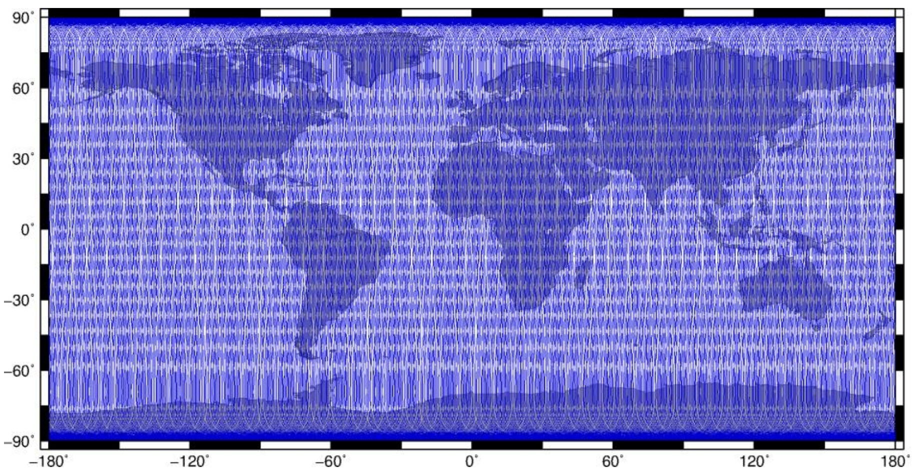
Coefficient difference



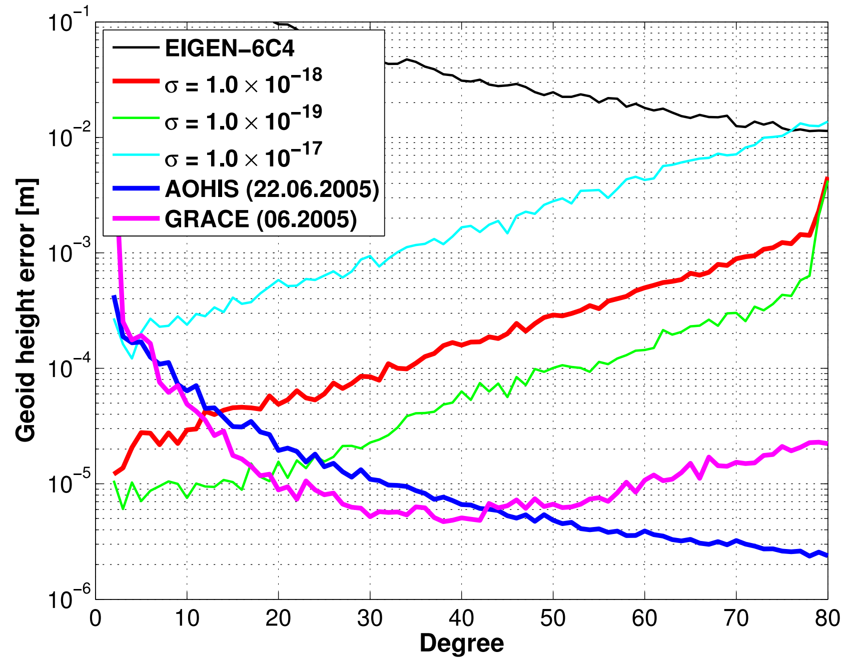
Potential for deriving time-variable signals

Simulated orbit for deriving monthly gravity field solutions:

- $h = 350$ km;
- $i = 89.5°$;
- $\alpha = 24, \beta = 377$ (repeat cycles)



Error degree variances



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Instrumentation

The cold atom interferometry (CAI) gravimeter

$$\Delta\varphi = -k_{eff} \cdot g \cdot T^2$$

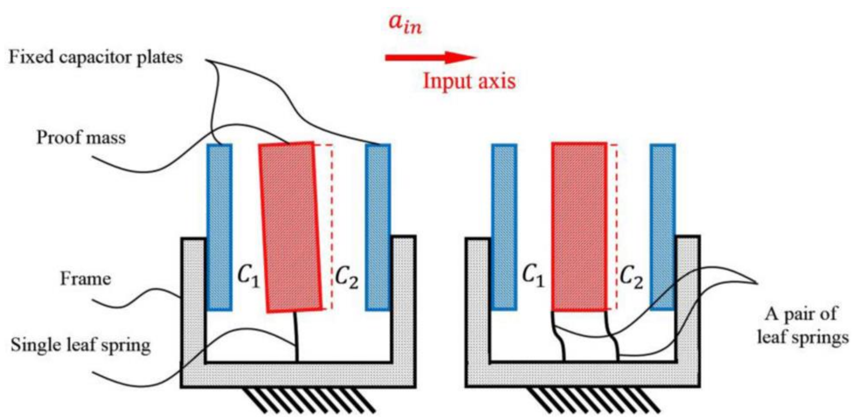


Figure is taken from S. Yan et al (2017).

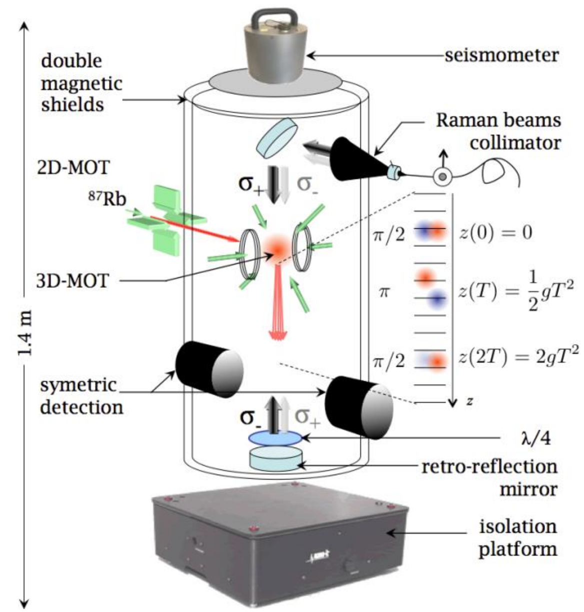


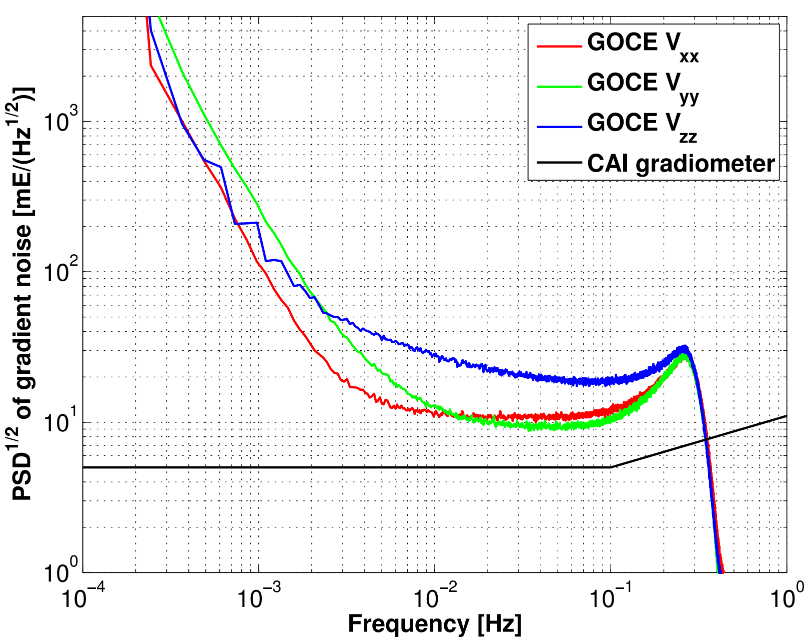
Figure is taken from SYRTE website.

electrostatic gravimeter → cold atom interferometry gravimeter

Performance

The CAI gradiometer has

- better sensitivity: **1.0 – 5.0** mE/sqrt(Hz);
- wide spectral range: white noise down to very low frequencies.



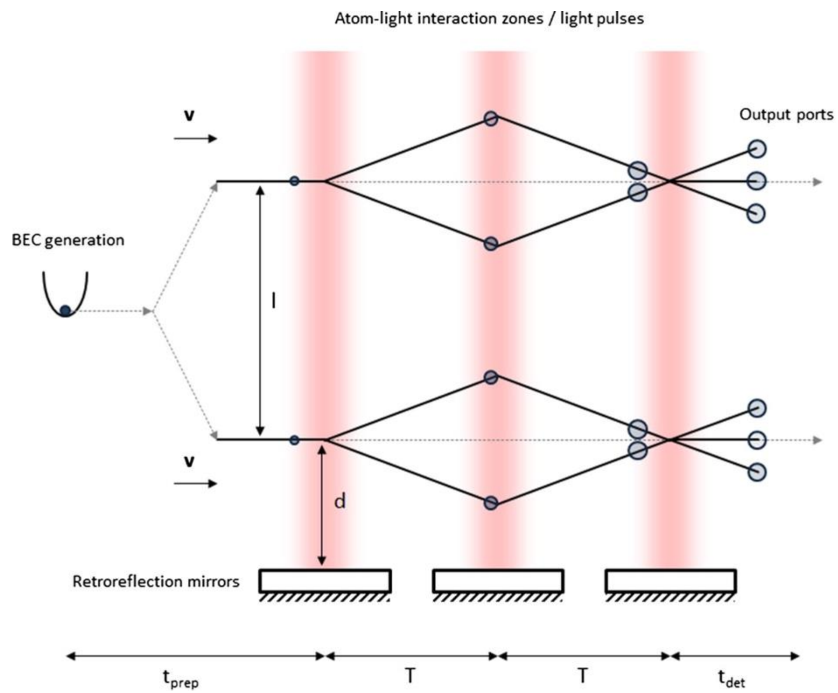


Figure is taken from O. Carraz et al. (2014).

Simulation

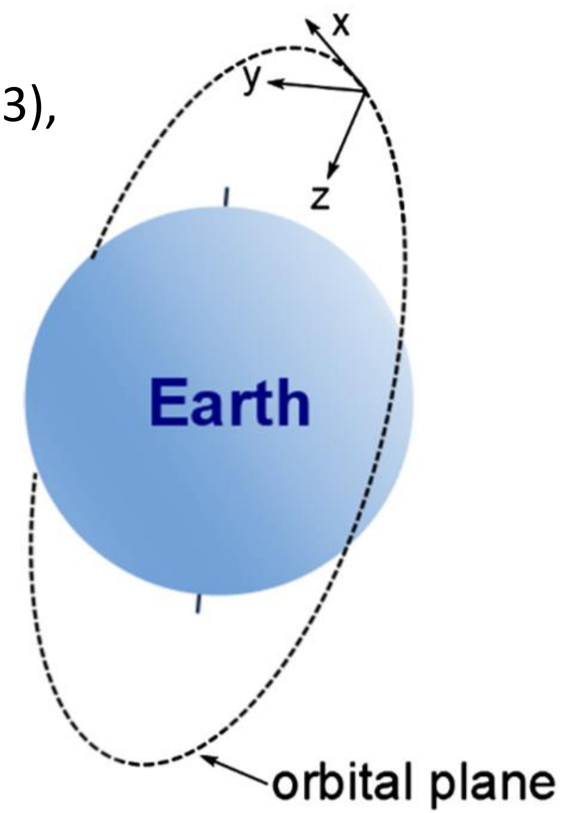
Input for simulations:

- Orbit: GOCE, 71 days (1st March – 10th May, 2013), altitude 239 km, 2 s;
- Model: Eigen-6c4, d/o 360;
- Noise: white, 5.0 mE/sqrt(Hz);

Two pointing modes:

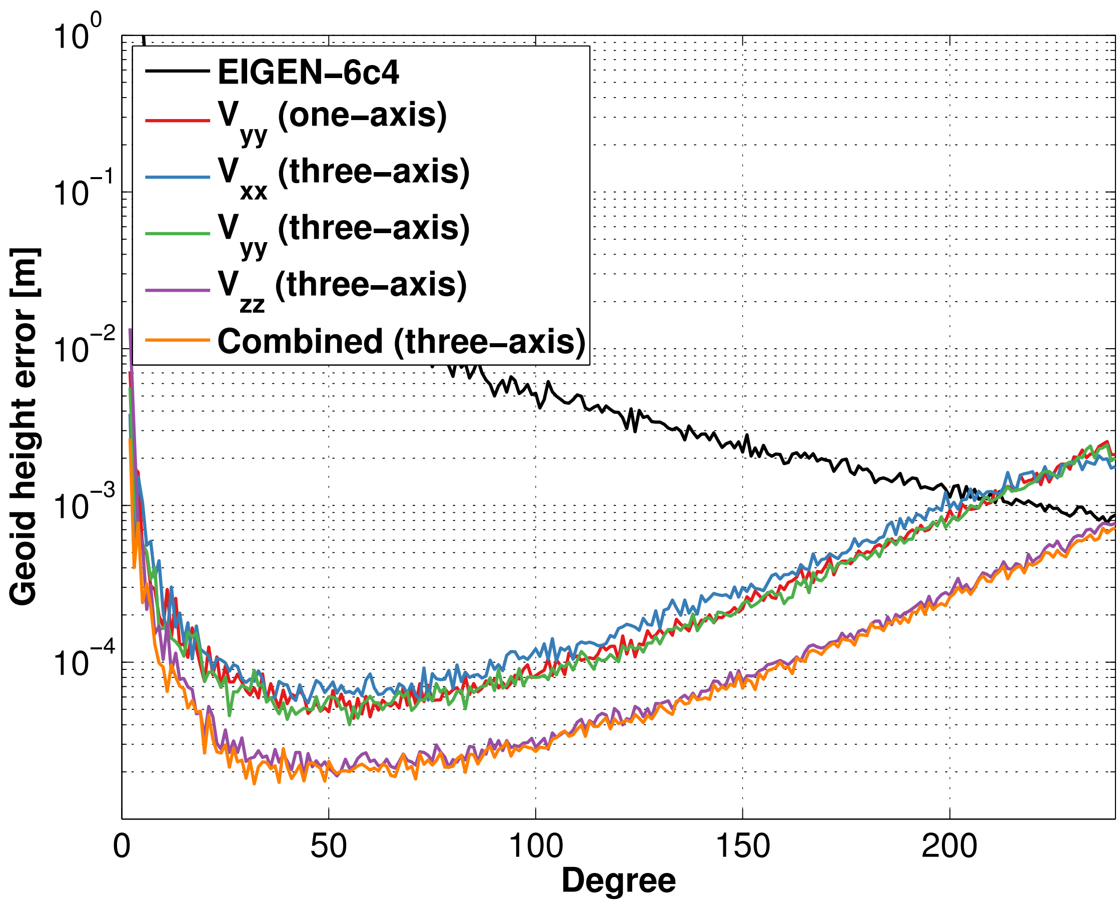
- Nadir case:
  - one axis: V_{yy}
  - three axis: V_{xx}, V_{yy} and V_{zz} (tilting mirror)
- Inertial case: V_{xx}, V_{yy} and V_{zz}

The model was recovered to d/o **240.**



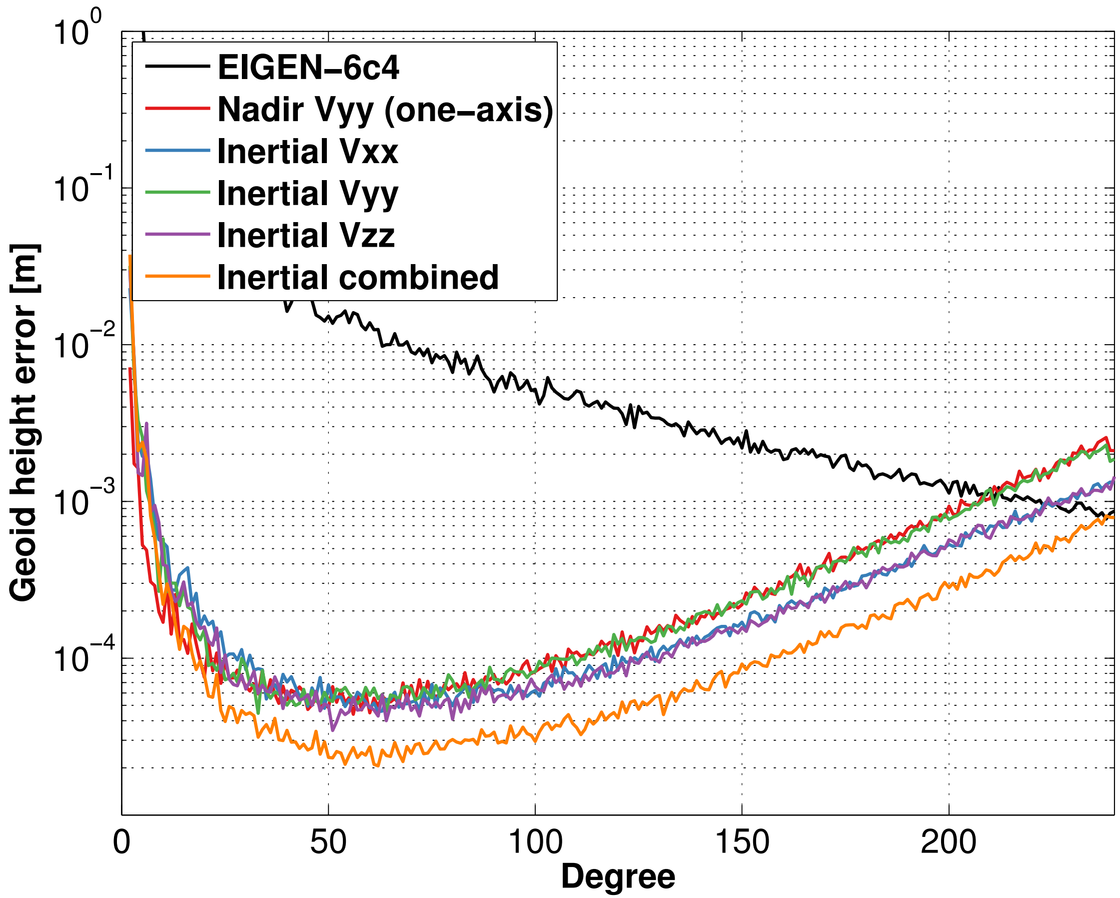
Results

Error degree medians for one-axis and three-axis nadir solutions



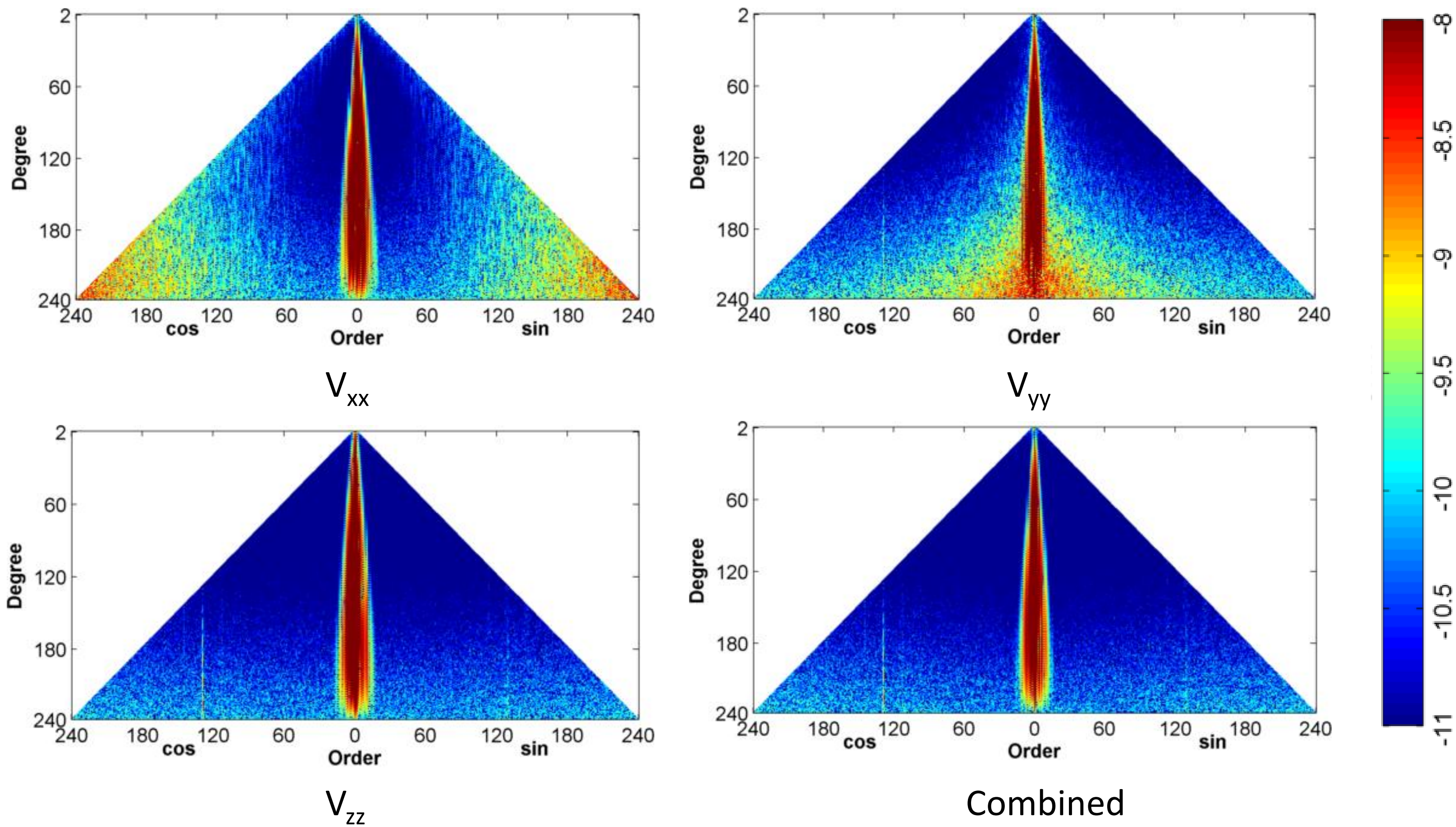
Results

Error degree medians for one-axis nadir and inertial solutions



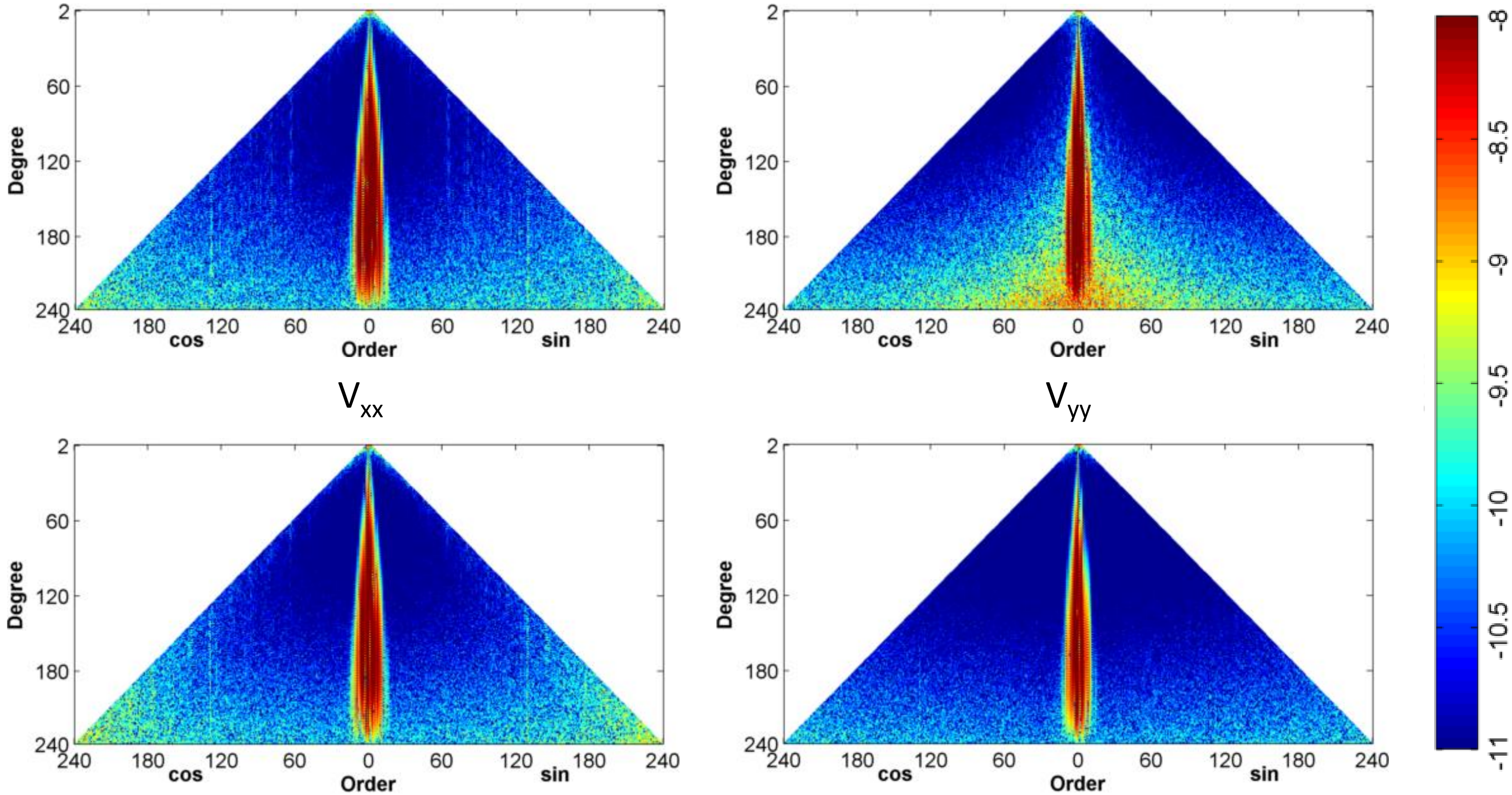
Results

Coefficient differences w.r.t. Eigen-6c4 (3-axis nadir case)



Results

Coefficient differences w.r.t. Eigen-6c4 (inertial case)



V_{xx} V_{yy}

V_{zz} Combined

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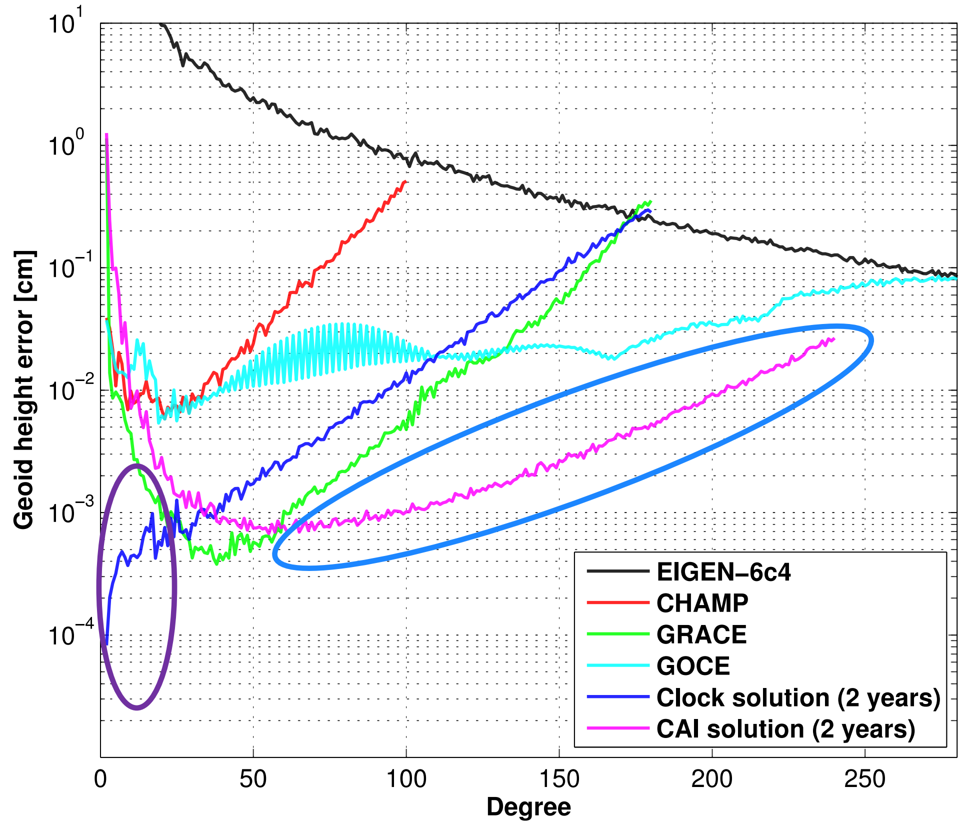
Combined analysis

We scaled the clock and three-axis CAI data using the $\sqrt{N}$ rule to adapt to a long timespan, and compared them with GRACE and GOCE models.

Error degree variances of coefficient difference w.r.t. Eigen-6c4



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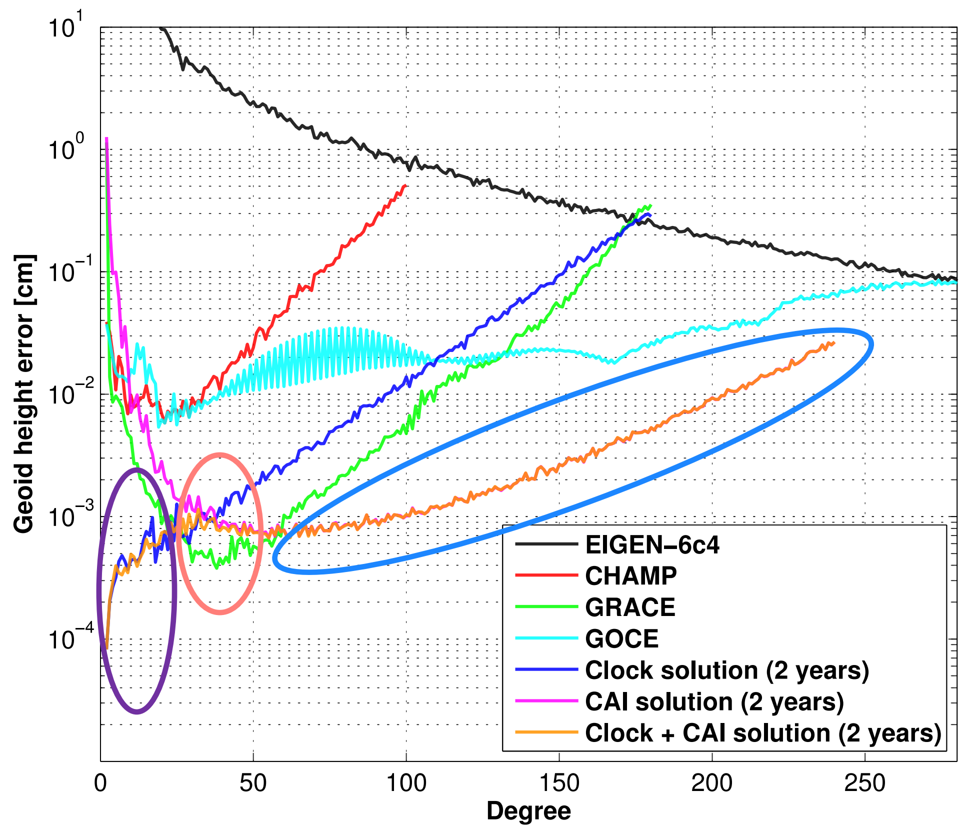


Zero-frequency shift for clocks in space

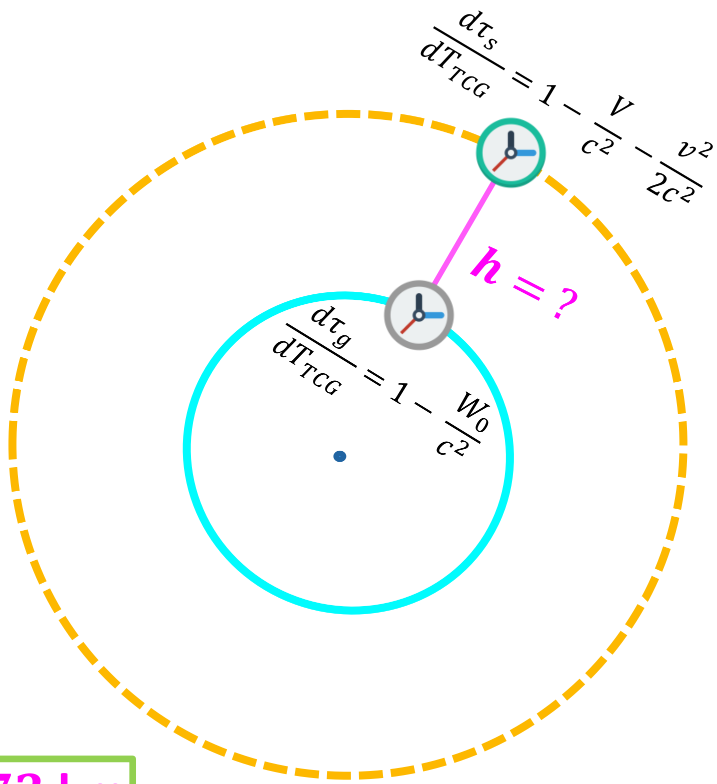
Comparison between clocks in space and on Earth's geoid

$$\frac{d\tau_s}{d\tau_g} = \frac{df_g}{df_s} = \frac{1 - \frac{V_s}{c^2} - \frac{v_s^2}{2c^2}}{1 - \frac{W_0}{c^2}}$$

where:

$$V_s \approx \frac{GM}{r}, v_s \approx \sqrt{\frac{GM}{r}}$$

$$W_0 = 62636853.4 \text{ m}^2/\text{s}^2$$



Supposing no-frequency shift:

$$\frac{d\tau_s}{d\tau_g} = 1 \Rightarrow \boxed{h = r - R = 3167.373 \text{ km}}$$

Zero-frequency shift for clocks in space

Zero-frequency shift surface

In principle, there exists a surface which is located at an altitude of about half the Earth's radius, where clocks tick at the same rate as those on the Earth's geoid.

This surface might be used

- to validate free-air link techniques (interlink between space-clocks, or link between clocks in space and on ground);
- as spatial reference surface (clocks there may replace those on ground?)
- …

Conclusions and future perspectives

Conclusions

- Clocks deliver scalar observations (less affected by attitude errors) and improve the long-wavelength gravity field (below D/O 30).
- Clocks show great potential to detect temporal gravity field signals at very low degrees;
- CAI gradiometry in 3-axis modes has the potential to outperform GOCE by about one order of magnitude.

Future perspectives

- Elaborate the procedure for frequency comparisons between clocks in space and on ground, and refine the noise model of the clock data;
- Study of CAI gradiometry in 3-axis mode for more realistic mission scenarios.

Thanks for your attention!

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