THE EUROPEAN GRAVIMETRIC QUASIGEOID EGG97 - AN IAG SUPPORTED CONTINENTAL ENTERPRISE - †

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ABSTRACT AND INTRODUCTION

In 1990, the Institut für Erdmessung (IfE), University of Hannover, started with the calculation of a gravimetric (quasi)geoid for Europe and the surrounding marine areas, operating as the computing center of the International Association of Geodesy (IAG) Geoid Commission. The requirements for accuracy and resolution were derived from the potential of GPS heighting and satellite altimetry, and defined at the “cm” to “dm” level over distances from a few km to some 1000 km, which requires a spatial resolution of a few km.

The data base established and continously extended and updated at IfE now includes several global gravity models, about 2.7 million (mainly point) gravity data, and about 700 million terrain heights. In some marine areas, gravity anomalies derived from ERS-1 satellite altimetry had to be included in the solution. All data sets were transformed to uniform standards in gravity, position and height.

Several quasigeoid solutions have been presented and discussed since 1990. The modeling strategy is based on the spectral combination technique in connection with the remove-restore procedure. The final solution combines the global Earth model EGM96 of NASA/NIMA with the high resolution gravity and terrain data stored in the IfE data base, including data from Russia and other Eastern European countries. The quasigeoid model is provided in a 1´ x 1.5´ and a 10´ x 15´ grid. In areas with a good data coverage and quality the accuracy estimates range from 1 ... 5 cm over 10 km to a few 100 km distance, and 5 ... 20 cm over a few 1000 km, respectively. Medium and long wavelength (larger than a few 100 km) errors have been found by comparisons with GPS/leveling control points, and are attributed mainly to errors of the global model, but also systematic errors in the gravity and GPS/leveling data are possible in some regions. The EGG97 quasigeoid model is now made available on a CD-ROM to users in geodesy, geophysics and engineering.

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1. COMPUTATION TECHNIQUE

The IfE gravity field modeling effort for Europe has concentrated on the calculation of height anomalies or quasigeoid undulations ζ. This has the advantage that only gravity field data observed at the Earth's surface and in its exterior enter into the calculations, while no assumptions about the gravity field in the Earth's interior are needed. Subsequently, a transformation from height anomalies ζ to geoid undulations N can be performed easily by introducing a density model.

The remove-restore technique is used to combine a high-degree spherical harmonic model and a digital terrain model (DTM) with terrestrial gravity field observations (point gravity data, etc.). For the field transformation from gravity to height anomalies, the least squares spectral combination technique is applied in order to reduce long-wavelength distortions, which may result from the use of Stokes's formula (see e.g. *Denker* et al. 1994). In the least squares spectral combination technique, instead of the Stokes kernel, a modified integral kernel

$$W(\psi) = \sum_{l=2}^{\infty} \frac{2l+1}{l-1} w_l P_l(\cos\psi) . \qquad (1)$$

is used (see e.g. *Wenzel* 1982). In Eq. (1) l is the degree, P_l are the Legendre polynomials, ψ is the spherical distance, and w_l are the spectral weights with

$$w_l = \frac{\sigma_l^2(\varepsilon_M)}{\sigma_l^2(\varepsilon_M) + \sigma_l^2(\varepsilon_{\Delta g})} . \qquad (2)$$

The w_l in Eq. (2) depend on the error degree variances of the potential coefficient model $\sigma_l^2(\varepsilon_M)$ and the gravity anomalies $\sigma_l^2(\varepsilon_{\Delta g})$, where the latter ones can be computed from the error covariance function of the terrestrial gravity data (see Eq. 3).

2. DATA DESCRIPTION

In the course of the European Geoid Project, about 2.7 million gravity data and about 700 million topographical height data have been included in the project data base. The gravity data coverage for the computation area is depicted in Figure 1, showing that land gravity data with a resolution of at least 10 km were attained for all European countries, while for some sea areas the coverage with terrestrial gravity data is still insufficient. Therefore, the ship data were merged with altimetrically derived gravity anomalies from ERS-1 (*Andersen* et al. 1996) in most parts of the European seas.

The digital terrain models were regridded to a common block size of 7.5″ x 7.5″ (or multiples of this block size), with existing gaps being filled by values from ETOPO5. Figure 2 depicts the coverage of high resolution DTM's stored in the IfE data base. Prior to utilizing all the collected data, a transformation to uniform standards in gravity, position and height was performed whenever this was possible. Furthermore, all data were validated using batch and interactive procedures developed at IfE (see also *Denker* et al. 1994).

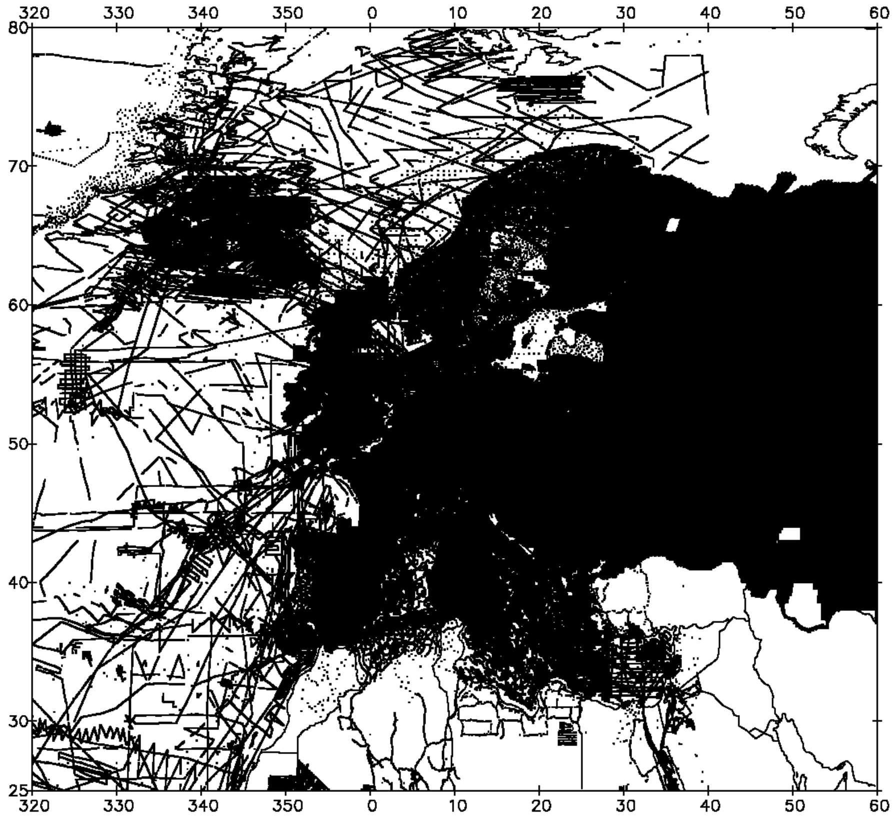

Fig. 1. Locations of gravity data stored in IfE data base. Status January 1997.

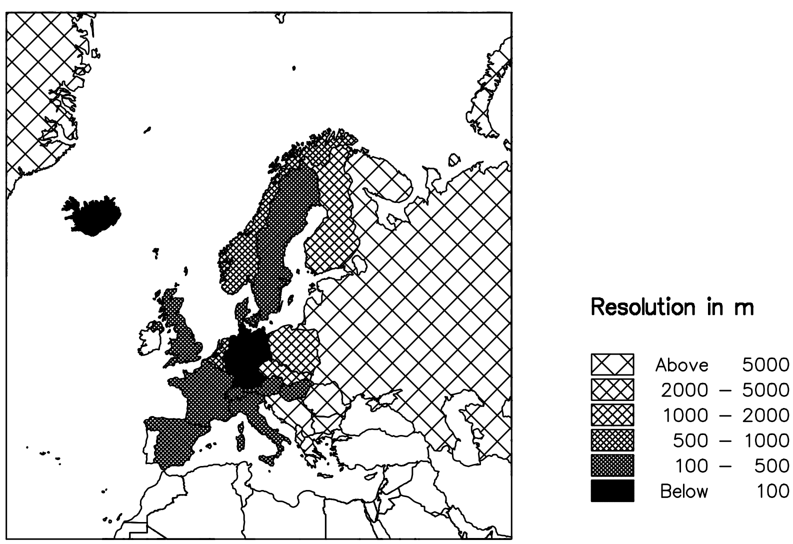

Fig 2. Digital terrain models stored in IfE data base. Status January 1997.

3. THE QUASIGEOID MODEL EGG97

In 1997, a new quasigeoid solution, EGG97, was computed for the whole of Europe. For the long wavelength gravity field information, the geopotential model EGM96 (*Lemoine* et al. 1997) was employed. The short wavelength gravity field components were modeled using the residual terrain model (RTM) reduction technique, where the reference topography was constructed from the DTM's using a 15´ x 15´ moving average filter. The terrain reductions for the gravity observations were computed by numerical integration techniques, while the terrain effects for the height anomalies (restore part) were computed by spherical 1D FFT techniques.

At first, residual gravity anomalies were computed and gridded by a fast least squares prediction technique onto a 1.0´ x 1.5´ grid covering the area from 25°N - 77°N and 35°W - 67.4°E. This yields 3,120 x 4,096 = 12,779,520 grid points. The field transformation from residual gravity to residual height anomalies was carried out using the spectral combination technique. The numerical evaluation of the integral formula was done by a 1D FFT technique in connection with a detailed/coarse grid approach to further speed up the computations. For the spectral combination technique the following error covariance function for the terrestrial gravity data was used:

$$\mathrm{cov}\left(\varepsilon_{\Delta g}, \varepsilon_{\Delta g}\right) = 4\left[\mathrm{mgal}^2\right] e^{-4\psi[^\circ]} . \quad (3)$$

This model uses correlated noise and was suggested and applied by *Weber* (1984). The spectral weights were derived on the basis of Eq. (2) using the above error covariance function for the terrestrial gravity data and the error degree variances from EGM96. It was decided to do the combination only up to degree 50, while between degrees 50 and 10000 (corresponding to the grid size used) the total weight was given to the terrestrial gravity data (w_l = 1.0). However, this does not imply that the global model EGM96 is completely disregarded above degree 50, as especially in areas with larger data gaps the high degree gravity information of the model is considered in the gridding process and thus practically taken over in the final quasigeoid model. A cosine tapering window was applied between degrees 10000 and 30000 in order to prevent oscillations of the integral kernel.

The major contribution to the final quasigeoid (internal name EGG97.03) comes from the spherical harmonic model EGM96 with values ranging from -43.3 m to +67.9 m and a standard deviation of 25.6 m. The standard deviations of the contributions from the DTM and the terrestrial gravity data are 0.03 m and 0.41 m, respectively. However, the maximum DTM effects are about 0.8 m, while the maximum effects of the terrestrial gravity data are 4.3 m. Furthermore, for the final EGG97 model, a zero-degree undulation correction of -0.5 m was considered, taking into account the results obtained by *Rapp* and *Balasubramania* (1992) as well as from comparisons with GPS/leveling data. A tidal correction (std.dev. 2.1 cm) was applied to refer the undulations to the zero tide system as recommended by IAG. The Molodensky correction terms have been neglected so far, which can obtain maximum values of 10 cm in the Alps and 1 cm in the highlands, respectively. Finally, geoid undulations were also derived based on a Bouguer plate model with constant density for the computation of the mean value of gravity. This corresponds to the so-called Helmert heights.

The spectral combination technique also permitted the derivation of error estimates for the resulting height anomalies. For the EGG97 model, $\sigma_{\Delta g}$ = 2 mgal gives standard deviations for height anomaly differences of 7.6 cm over 100 km and 12.7 cm over 1000 km, respectively. A more optimistic error estimate for the terrestrial gravity data of $\sigma_{\Delta g}$ = 1 mgal gives standard deviations of 3.9 cm over 100 km and 7.6 cm over 1000 km, respectively. The corresponding values for $\sigma_{\Delta g}$ = 4 mgal are 15.2 cm and 23.9 cm, respectively.

4. Evaluation of the Quasigeoid Model EGG97

The quasigeoid model EGG97 was verified by means of satellite altimeter data and GPS/leveling data. However, in this paper we will restrict the quality assessment of EGG97 to the comparison with three GPS/leveling data sets covering small (NDS92, Lower Saxony,

Table 1: Statistics of the differences from the comparison of recent quasigeoid solutions with different GPS/leveling data sets. Units are meters.

Quasigeoid Solution	Bias Fit		Bias + Tilt Fit	
	RMS	Max.	RMS	Max.
NDS92 (Lower Saxony; 41 stations)				
EGG1 (1982)	0.108	0.368	0.061	0.221
EAGG1 (1983)	0.069	0.171	0.063	0.162
OSU91A (1991)	0.195	0.912	0.143	0.651
EGM96 (1996)	0.148	0.462	0.136	0.567
EGG96 (1996)	0.039	0.090	0.015	0.032
EGG97 (1997)	0.038	0.090	0.013	0.033
RBF (France; 965 stations)				
EGG1 (1982)	0.664	2.937	0.381	2.079
EAGG1 (1983)	0.460	1.882	0.387	1.803
OSU91A (1991)	0.374	2.240	0.337	2.027
EGM96 (1996)	0.369	2.025	0.301	1.698
EGG96 (1996)	0.106	0.341	0.070	0.407
EGG97 (1997)	0.128	0.353	0.080	0.484
European GPS Traverse (67 stations)				
EGG1 (1982)	0.605	1.351	0.275	0.778
EAGG1 (1983)	0.240	0.607	0.175	0.501
OSU91A (1991)	0.319	0.926	0.260	0.795
EGM96 (1996)	0.304	0.875	0.251	1.114
EGG96 (1996)	0.307	0.851	0.157	0.436
EGG97 (1997)	0.294	0.793	0.175	0.470

300 km extension), medium (RBF, France, 1000 km extension) and large scales (European GPS traverse, 3000 km length). Statistics of the discrepancies between the GPS/leveling data and some recent quasigeoid solutions are given in Table 1. The comparisons were always done using a bias fit as well as a bias and tilt fit in order to account for inaccuracies in the absolute positioning and for long wavelength errors of all data sets involved (GPS, leveling, quasigeoid).

From Table 1, it becomes clear that the new quasigeoid solution EGG97 is a significant improvement over the older models EGG1 (*Torge* et al. 1982) and EAGG1 (*Brennecke* et al. 1983), with the RMS discrepancies decreasing by a factor of 2 to 5. Moreover, in most cases one can observe a significant improvement for the bias and tilt fit versus the bias fit, thus indicating that small long wavelength discrepancies exist between the gravimetric and the GPS/leveling results (magnitude 0.1 to 0.5 ppm). The EGG96 (*Denker* et al. 1997) and EGG97 models give comparable results. The only difference between these models is the use of a different global model (EGM96 versus JGM3_OSU91A) and the use of one new gravity data source in EGG97.

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