EUROPEAN GRAVIMETRIC GEOID: STATUS REPORT 1994 †

Heiner Denker, Dirk Behrend, Wolfgang Torge
Institut für Erdmessung, University of Hannover, Nienburger Straße 6
D-30167 Hannover, Federal Republic of Germany

ABSTRACT

This paper describes the current status of the European quasigeoid calculation performed at the Institut für Erdmessung (IfE). Emphasis is put on the progress made in data collection (gravity and terrain) as well as on the refinement of the computation strategies (spectral combination). The resulting quasigeoid models, which were evaluated by comparisons with GPS/leveling and Topex/Poseidon altimeter data, show an accuracy of ±1...5 cm over 10 to a few 100 km distance, and ±5...20 cm over a few 1000 km distance, respectively (best solution). At present, long wavelength errors of the global gravity models and the terrestrial gravity data pose the major problems.

1. INTRODUCTION

Till the beginning of the 1980's geoid calculations for the whole of Europe were limited to a few decimeters accuracy with a maximum spatial resolution of some 20 km. The following decade brought along major changes through improved modeling techniques, the availability of high-resolution gravity field data sets, and substantially increased computing power. Thus, regional geoid determinations with an accuracy improvement up to an order of magnitude became feasible. Apart from this, also the need for a "cm"-geoid arose, as the Global Positioning System (GPS), which is now fully operational, provides ellipsoidal heights at the cm level over distances from a few km to a few 1000 km. In order to make use of this high accuracy level, geoid resp. quasigeoid undulations at the same level of accuracy are required. IfE, therefore, is working on the determination of a high-precision and high-resolution European quasigeoid model under the auspices of the International Geoid Commission. Several preliminary solutions were presented at different places since the initiation of the project in 1990 (Vienna, Prague, Wiesbaden, Beijing). The final gravimetric quasigeoid solution is planned to be presented at the IUGG General Assembly in Boulder, 1995.

2. COMPUTATION TECHNIQUES

In our gravity field modeling effort for Europe we are primarily interested in the calculation of height anomalies respectively quasigeoid undulations ζ. This has the advantage that only gravity field data observed at the Earth's surface and in its exterior enter into the calculations, while no assumptions about the gravity field in the Earth's interior are needed. The height anomaly is related to the disturbing or anomalous potential T, playing the central role in gravity field modeling, by Bruns equation

$$\zeta = \frac{T}{\gamma}, \tag{1}$$

† In: H. Sünkel, I. Marson (Eds.), Gravity and Geoid, Joint Symp. of the Internat. Gravity Comm. and the Internat. Geoid Comm., Graz, Austria, Sept. 11-17, 1994, IAG Symposia, 113:423-432, Springer-Verlag, 1995.

where γ is the normal gravity. If desired, a subsequent transformation from height anomalies ζ to geoid undulations *N* can be performed easily by introducing a density model:

$$N = \zeta + \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H \; . \tag{2}$$

Here $\bar{g}$ is the mean value of gravity depending on the density model, $\bar{\gamma}$ is the mean value of normal gravity, and *H* is the orthometric height (for more details see e.g. Torge 1991).

Our basic gravity field modeling strategy is based on the remove-restore technique. In this procedure a high-degree spherical harmonic model and a digital terrain model (DTM) are combined with terrestrial gravity field observations (point gravity data, etc.). In this procedure, a residual potential function and the corresponding residual observations are computed first by

$$T_3 = T - T_1 - T_2 \; , \tag{3}$$

$$L_i(T_3) = L_i(T) - L_i(T_1) - L_i(T_2) \; , \tag{4}$$

where T_1 and T_2 are the components associated with the spherical harmonic model and the DTM (or more generally the mass model), T_3 is the residual potential, and L_i is a linear functional.

The modeling techniques are then applied to the residual data, and finally the effect of the spherical harmonic model and the DTM are added back to all predicted quantities yielding in

$$L_j(\hat{T}) = L_j(T_1) + L_j(T_2) + L_j(\hat{T}_3) \; . \tag{5}$$

The remove-restore technique was used successfully in the past in connection with least squares collocation and integral formulas. Both methods give comparable results (see e.g. Denker 1988, Bašić 1989), but the use of integral formulas together with FFT is much more efficient. For the computation of continental-scale geoid/quasigeoid models the use of integral formulas together with FFT is the only practicable technique to date.

Currently our main interest is the calculation of a new quasigeoid model for Europe based on point and mean gravity data, a high-resolution spherical harmonic model and a DTM. The fundamental equation for this calculation is the Stokes's resp. Molodensky's equation:

$$\zeta = \frac{R}{4\pi\gamma} \iint_\sigma \Delta g \; S(\psi) \, d\sigma + c_M \; , \tag{6}$$

$$S(\psi) = \sum_{l=2}^{\infty} \frac{2l+1}{l-1} P_l(\cos\psi) \; , \tag{7}$$

where *R* is an average Earth radius, Δg is the gravity anomaly, $S(\psi)$ is the Stokes function, P_l are the Legendre polynomials, and c_M are the Molodensky correction terms, which are neglected in our computations. Equations (6) and (7) have been applied to unreduced free-air gravity anomalies as well as to residual gravity anomalies (4), both referring to the surface of the earth. However, when working with residual data (reduced for the effect of a global model and a mass model) in connection with the remove-restore procedure, the use of Stokes equation implies that the complete spectrum of the height anomalies (degree 2 to infinity) is computed from the terrestrial gravity anomalies in the integration area augmented by the global model values outside this area. In case that long wavelength discrepancies exist between the terrestrial gravity data and the global model, the application of (6) and (7) will lead to an unreasonable distortion of the long wavelength components of the global model. Such effects were clearly seen in our previous solutions when comparing the results with satellite altimeter and GPS/leveling data. We found very long wavelength discrepancies and strong tilts (with a magnitude of several meters), which turned out to be completely unrealistic.

To overcome this problem we decided to apply the least squares spectral combination technique going back to Moritz (1976) as well as Sjöberg (1981) and Wenzel (1982). Here the final height anomalies are obtained according to equation (5) by

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3 . \tag{8}$$

The effect of the local gravity data is obtained by the following equation:

$$\zeta_3 = \frac{R}{4\pi\gamma} \iint_\sigma (\Delta g - \Delta g_1 - \Delta g_2) W(\psi)\, d\sigma , \tag{9}$$

$$W(\psi) = \sum_{l=2}^{\infty} \frac{2l+1}{l-1} w_l P_l(\cos\psi) , \tag{10}$$

where $\Delta g_3 = \Delta g - \Delta g_1 - \Delta g_2$ are the residual gravity anomalies, $W(\psi)$ is the modified integration kernel, and w_l are the spectral weights. In (10) the w_l determine how much signal is taken from the terrestrial gravity data at a certain degree l, being dependent on the height anomaly error degree variances of the potential coefficients $\sigma_l^2(\varepsilon_1)$ and the gravity anomalies $\sigma_l^2(\varepsilon_{\Delta g})$:

$$w_l = \frac{\sigma_l^2(\varepsilon_1)}{\sigma_l^2(\varepsilon_1) + \sigma_l^2(\varepsilon_{\Delta g})} . \tag{11}$$

In the above equation the $\sigma_l^2(\varepsilon_{\Delta g})$ can be computed from the error covariance function of the terrestrial gravity data (see e.g. Wenzel 1982).

Finally it should be noted that the above equations (6)-(7) and (8)-(10) assume that the “true” geocentric gravitational constant of the Earth GM is equal to the corresponding value of the reference ellipsoid GM^0, and that the gravity potential of the geoid W_0 is equal to the gravity potential of the surface of the reference ellipsoid U_0. If such differences exist, this leads to the so-called zero order undulation

$$\zeta_0 = \frac{GM - GM^0}{r\gamma} - \frac{W_0 - U_0}{\gamma} , \tag{12}$$

which has to be added to (6) and (8) respectively. If this basically constant term is neglected, the resulting height anomalies refer to an ideal ellipsoid with the properties $GM=GM^0$ and $W_0=U_0$, but whose dimensions (equatorial radius a) are not precisely known in terms of numerical values. This is a key problem since the ellipsoidal heights from, e.g., GPS refer to a specific reference ellipsoid (for more details see e.g. Rapp and Balasubramania 1992). This problem is usually overcome by considering a bias term in the comparison of GPS/leveling and gravimetric results. In many cases additional tilts in north-south and east-west direction are considered to model long wavelength errors of the gravimetric results and inaccuracies in the absolute positions of GPS (see below).

3. Data Description

This section gives an overview on the data sets currently included in the gravity field data base at IfE. The data base comprises about 1.5 million gravity data and 650 million topographical data. Figure 1 gives a graphical representation of the gravity data coverage in the computation area. From the figure it becomes clear that the coverage with gravity observations is not sufficient for some marine areas as well as for the former Soviet Union. Therefore, we decided to use altimetrically derived gravity anomalies from Bašić and Rapp (1992) for the marine areas with insufficient data coverage. For the area of the former Soviet Union we used the 1° x 1° data set from Bureau Gravimétrique, being the only source of information available at present.

Prior to utilizing these data in the quasigeoid computation, a transformation into a common reference system (IGSN 71, GRS 80 normal gravity formula) was carried out. Furtheron, all data were validated using batch and interactive procedures developed at IfE. The basic principle of this software is to compare each gravity observation with a value predicted from the adjacent stations. Unrealistic values, showing large discrepancies, were then excluded from the quasigeoid calculations.

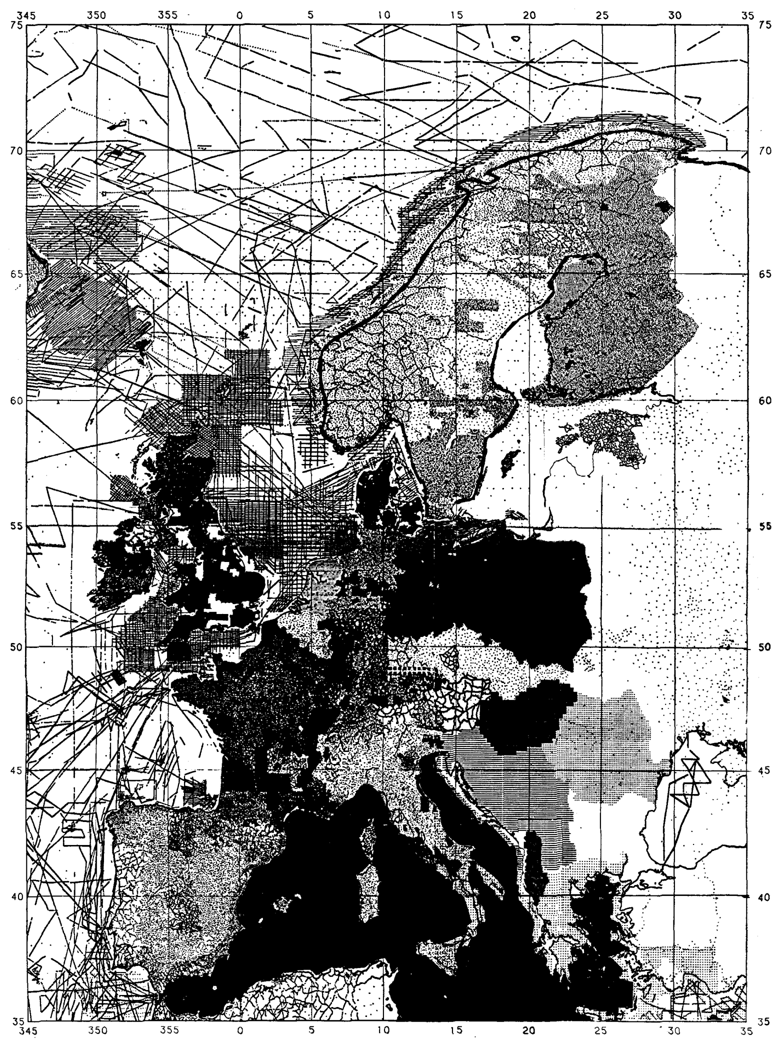

Fig. 1. Locations of point gravity data stored in the IfE data base (status September 1994).

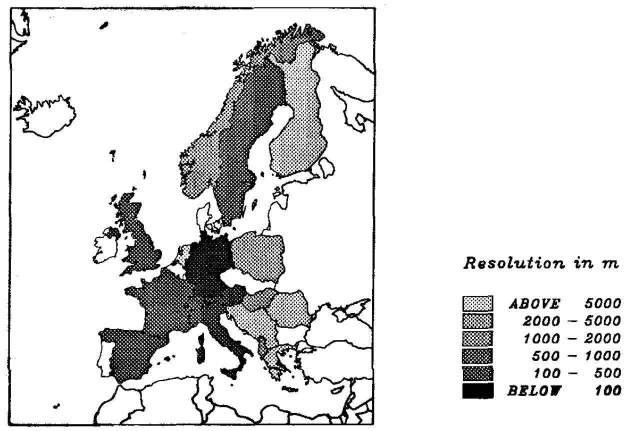

Fig. 2. Digital terrain models stored in the IfE data base (status September 1994).

The terrain data were subject to a similar validation process by comparing each elevation with adjacent values. Here, unlike the gravity data, unrealistic values were replaced by interpolated or apparently correct values (as is e.g. the case for intermixed numbers). Smaller gaps were filled through interpolation, larger gaps and blank areas were allocated values from ETOPO5. Finally, the digital terrain models were regridded to a common block size of $7.5'' \times 7.5''$ (or multiples of this) and transformed to the WGS 84 geocentric reference system. Figure 2 depicts the coverage with high resolution DTM's used for the present quasigeoid solution. Still missing are models for Iceland, Ireland, Portugal, Belgium, Luxembourg, Czech Republic, Slovak Republic, Denmark, Bulgaria, and states of the former Soviet Union.

4. THE 1994 QUASIGEOID SOLUTIONS

In 1994 six new quasigeoid solutions were computed for entire Europe based on Stokes equation and the spectral combination technique in connection with the remove-restore procedure. For the long wavelength gravity field information the spherical harmonic model OSU91A complete to degree and order 360 (Rapp et al. 1991) was employed. The short wavelength gravity field components were modeled using the residual terrain model (RTM) reduction technique according to Forsberg and Tscherning (1981), where the reference topography was constructed by a 30' x 30' moving average filter. The residual gravity anomalies were gridded by a fast least squares prediction technique onto a $1.0' \times 1.5'$ grid covering the area from 25°N - 75°N and 35°W - 67.4°E. This yields $3{,}000 \times 4{,}096 = 12{,}288{,}000$ grid points. The field transformation from residual gravity to residual height anomalies was carried out using equations (6)-(7) and (8)-(10). The practical evaluation of these integral formulas was done by a 1D FFT technique suggested by Haagmans et al. (1993) in connection with a detailed/coarse grid approach to further speed up the computations. The major advantage of this procedure is that an exact evaluation of the integrals on the sphere is possible (without any periodicity effects of FFT).

Table 1. The 1994 quasigeoid solutions.

Quasigeoid Solution	Description	Mean of Residual Gravity Anomalies Subtracted
94.01	Spectral Combination #1 (SC1)	no
94.02	Spectral Combination #2 (SC2)	no
94.03	Stokes	no
94.04	Spectral Combination #1 (SC1)	yes
94.05	Spectral Combination #2 (SC2)	yes
94.06	Stokes	yes

A summary of the six new quasigeoid solutions is given in table 1. For the spectral combination technique two different weighting sets were used. Both of them depend on the following error covariance function for the terrestrial gravity data:

$$\operatorname{cov}\left(\varepsilon_{\Delta g}, \varepsilon_{\Delta g}\right) = 16\left[\mathrm{mgal}^2\right] e^{-4\psi[^\circ]}. \tag{13}$$

This model uses correlated noise and was suggested and applied by Weber (1984). The two sets of spectral weights were derived on the basis of equation (11) using the above error covariance function for the terrestrial gravity data and the error degree variances from OSU91A (SC1) resp. from the underlying satellite-only model, being GEM-T2 (SC2). It was decided to do the combination only up to degree 50, while between degrees 50 and 10000 (corresponding to the grid size used) the complete gravity field information was taken from the terrestrial gravity data (w_l = 1.0). A cosine tapering window was applied between degrees 10000 and 30000. This turned out to be necessary because otherwise the integral kernel started to oscillate. The spectral weights as well as the corresponding integral kernels are shown in figure 3. As expected, the integral kernel SC1 goes faster to zero than SC2 because less weight is put on the long wavelength gravity field components.

The use of the spectral combination technique also permitted us to derive error estimates for the resulting height anomalies resp. differences thereof (see e.g. Wenzel 1982). For SC1 ($\sigma_{\Delta g}$ = $\pm$4 mgal) we get standard deviations for height anomaly differences of $\pm$15 cm over 100 km and $\pm$25 cm over 1000 km distance, respectively. In case of a more optimistic error estimate for the terrestrial gravity data ($\sigma_{\Delta g}$ = $\pm$1 mgal) we get standard deviations of $\pm$4 cm over 100 km and $\pm$12 cm over 1000 km distance, respectively. When looking at the GPS/leveling comparisons, the latter estimates appear to be more realistic (at least over shorter distances).

For all 6 quasigeoid solutions a statistics of the individual components ζ_1, ζ_2, ζ_3 is provided in table 2. Solutions 1-3 differ from 4-6 in the handling of the mean value of the residual gravity anomalies (see table 1). While the subtraction of the mean value of the residual anomalies (statistics of Δg_3: mean=0.17 mgal, std.dev.=15.48 mgal, min.=-148.05 mgal, max.= 169.82 mgal) has practically no effect on the spectral combination solutions, a significant effect can be seen for the Stokes solution. Although the mean value of the residual gravity data is very small (0.17 mgal), the mean value of the residual height anomalies changes by more than 30 cm. From table 2 it can also be observed that the standard deviations of the residual height anomalies are increasing from solution 1 to 3 and 4 to 6. However, this is expected due to the handling of the long wavelength gravity field components. Noteworthy are also the maximum corrections to OSU91A, exceeding 5 meters in areas where no data went into the model.

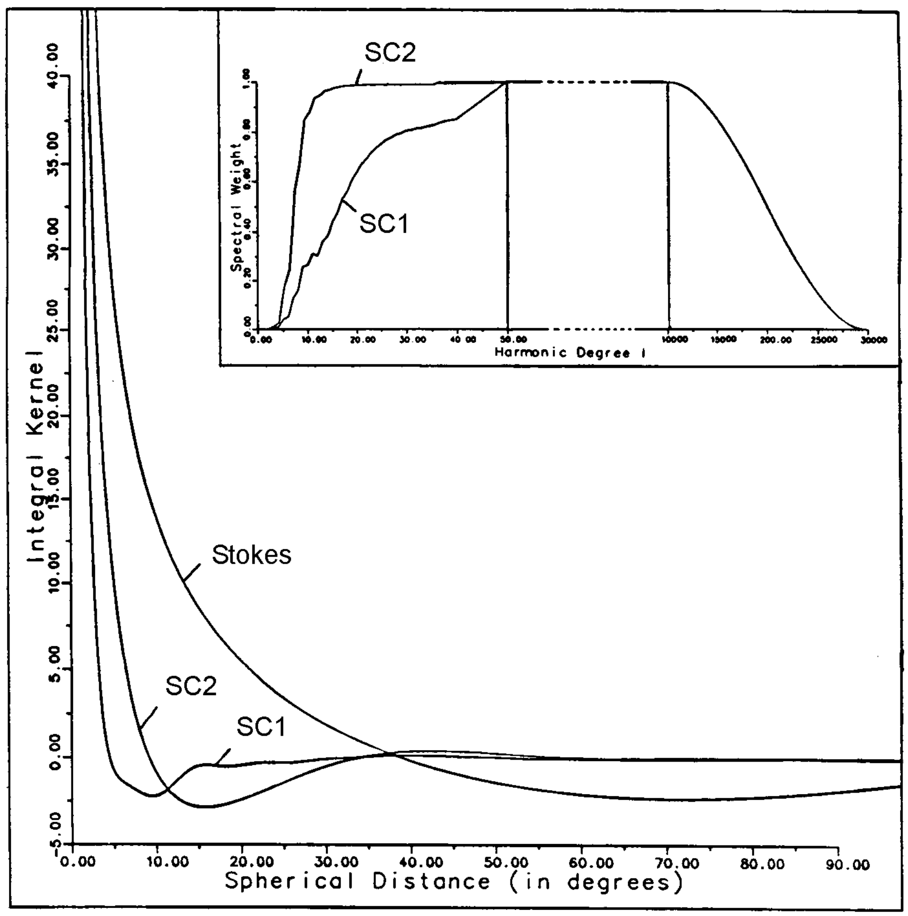

Fig. 3. Spectral weights and the corresponding integral kernels.

Table 2. Statistics of the 1994 quasigeoid solutions. Units are meters.

Parameter	Mean	Std.Dev.	Min.	Max.
ζ_1 (OSU91A)	28.411	25.762	-41.921	+68.016
ζ_2 (RTM 30'x30')	0.097	0.090	-0.755	+1.624
ζ_3 94.01 (SC1)	0.000	0.730	-9.787	+6.810
ζ_3 94.02 (SC2)	0.002	0.980	-9.752	+7.324
ζ_3 94.03 (Stokes)	0.396	1.669	-8.345	+8.779
ζ_3 94.04 (SC1)	-0.004	0.730	-9.801	+6.797
ζ_3 94.05 (SC2)	-0.008	0.977	-9.780	+7.303
ζ_3 94.06 (Stokes)	0.070	1.671	-8.595	+8.489

5. Evaluation of the 1994 Quasigeoid Solutions and Conclusions

The evaluation of the new quasigeoid solutions was carried out by means of satellite altimeter data and GPS/leveling data. At first, we will discuss the results using satellite altimeter data from the Topex/Poseidon (T/P) mission. In our comparisons we did not use a model for the dynamic sea surface topography as at present no reliable model is available for the European seas. Thus the sea surface heights from the GDRs (T/P reference system) were directly compared with the gravimetrically determined height anomalies (referring to an ideal Earth ellipsoid, see above). A statistics of the discrepancies, reflecting in principal the dynamic topography, is given in table 3 for repeat cycle #21, being a typical example. Please note that discrepancies exceeding 3σ were excluded from the comparisons. From graphical displays of the differences and also from the mean and RMS values given in table 3 it becomes clear that only solution 94.01 (and 94.04) give reasonable results. All other solutions (especially Stokes) show rather big long wavelength distortions which cannot be explained by dynamic topography. Thus we conclude that solution 94.01 is the best quasigeoid model.

Table 3. Statistics for the comparisons with Topex/Poseidon altimeter data. Units are meters.

Quasigeoid Sol.	# points	Mean	RMS
OSU91A	28,524	0.058	0.441
94.01 (SC1)	28,381	0.129	0.445
94.02 (SC2)	28,290	-0.013	0.662
94.03 (Stokes)	28,425	-0.663	1.360
94.04 (SC1)	28,379	0.120	0.443
94.05 (SC2)	28,277	-0.037	0.665
94.06 (Stokes)	28,356	-1.037	1.555

Further evaluations of the new quasigeoid solutions were done using a number of GPS/leveling data sets. The results based on three of these data sets will be discussed in the following. A statistics of the discrepancies is given in table 4. The comparisons were always done using a bias fit as well as a bias and tilt fit for reasons explained already in section 2. In all cases we see a significant improvement for the new solutions as compared to the existing solutions EGG1 (Torge et al. 1982) and EAGG1 (Brennecke et al. 1983). Furthermore, in most cases the new solution 94.01 yields the best results, which is in complete agreement with the altimeter comparisons. For the first and more local GPS/leveling data set for Lower Saxony, Germany (extension about 300 km), we get an RMS discrepancy of ± 0.065 m for the bias fit and ± 0.015 m for the bias and tilt fit using solution 94.01. Small but significant tilts (0.7 ppm) exist in this comparison, which start to decrease when taking more long wavelength information from the terrestrial gravity data, thus indicating problems in the global model (see statistics of the bias fit for solutions 94.01-94.03). For the European GPS traverse with a length of about 3000 km running from Austria to northern Norway we can also observe a small improvement in the bias and tilt fit when putting more weight on the terrestrial gravity data (± 0.160 versus ± 0.129 for SC1 and SC2 respectively). A graphical display of the comparison results for the GPS traverse is shown in figure 4.

Table 4. Statistics of the comparison of selected quasigeoid solutions with different GPS/leveling data sets. Units are meters.

GPS/Leveling Data Set	Quasigeoid Solution	Bias Fit		Bias + Tilt Fit	
		RMS	Max.	RMS	Max.
Lower Saxony	EGG1	0.107	0.378	0.062	0.234
1992 Data	EAGG1	0.071	0.192	0.066	0.179
41 Stations	93 (Stokes)	0.039	0.093	0.015	0.039
	94.01 (SC1)	0.065	0.146	0.015	0.041
	94.02 (SC2)	0.051	0.122	0.014	0.037
	94.03 (Stokes)	0.027	0.053	0.014	0.034
European GPS	EGG1	0.606	1.356	0.274	0.768
Traverse	EAGG1	0.241	0.611	0.175	0.503
67 Stations	93 (Stokes)	0.235	0.719	0.118	0.319
	94.01 (SC1)	0.224	0.660	0.160	0.439
	94.02 (SC2)	0.343	0.944	0.129	0.338
	94.03 (Stokes)	0.231	0.642	0.138	0.350
EUREF	EGG1	0.864	2.381	0.794	2.376
33 Stations	EAGG1	0.756	1.603	0.702	1.527
	93 (Stokes)	0.636	1.936	0.526	1.822
	94.01 (SC1)	0.429	1.059	0.291	0.611
	94.02 (SC2)	0.759	2.341	0.557	1.898
	94.03 (Stokes)	0.668	2.001	0.653	2.362

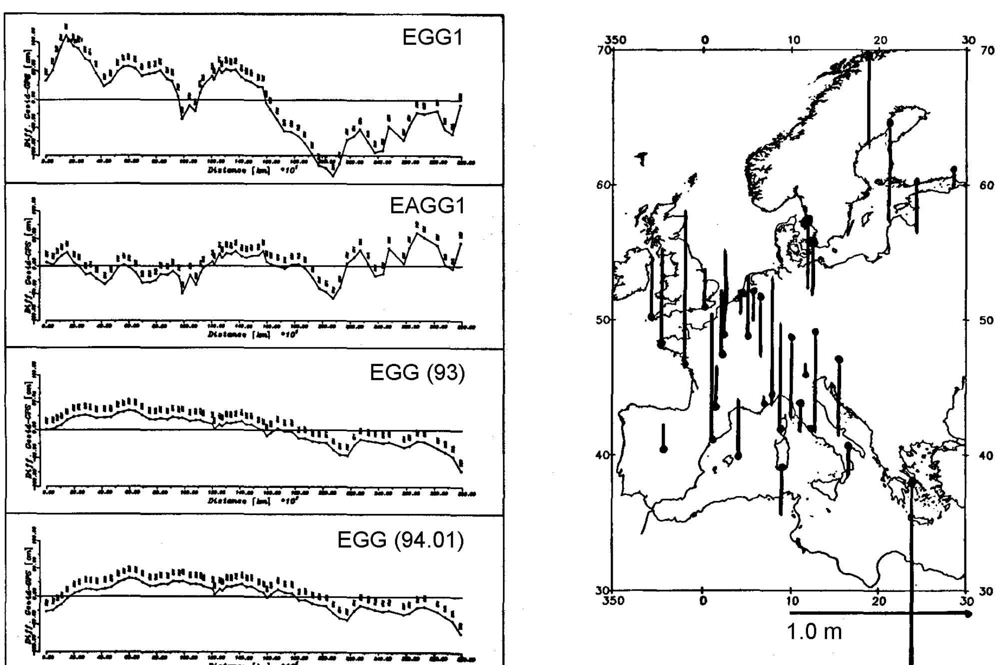

Fig. 4. Comparison of the 94.01 quasigeoid solution with GPS/leveling data from the European GPS traverse (left part) and the EUREF campaign (right part). A constant bias is subtracted.

While for the Lower Saxony and the GPS traverse data set strict normal heights were available, unfortunately this is not the case for EUREF. Here are only preliminary leveling heights without clear information on the height system and datum at our disposal. This comparison is therefore very preliminary and more work is necessary to transform all heights to a common height reference system. The discrepancies for the bias fit are shown in figure 4, and we can observe very significant offsets between different countries (especially Germany and France).

To conclude, significant progress was made since the initiation of the geoid project in 1990 regarding the collection of gravity and terrain data, the computation algorithm (spectral combination versus Stokes) and the evaluation of the results (use of GPS/leveling and Topex/Poseidon data). For areas with a good coverage and accuracy of the gravity and terrain data, the accuracy of the best solution is estimated as ±1...5 cm over 10 to a few 100 km distance, and ±5...20 cm over a few 1000 km distance, respectively. Problems that need to be further studied in the future concern long wavelength errors of the global gravity models and the terrestrial gravity data. Furthermore, some data gaps are still existing, but hopefully they can be filled before the project ends in 1995.

ACKNOWLEDGMENTS

The authors are grateful to all individuals and agencies providing data for this project. A complete list of all data contributors will be given in the final report.

REFERENCES

Bašić, T., R.H. Rapp (1992). Oceanwide prediction of gravity anomalies and sea surface heights using Geos-3, Seasat, and Geosat altimeter data and ETOPO5U bathymetric data. Dept. Geod. Sci. and Surv., Rep. 416, Columbus.

Bašić, T. (1989). Untersuchungen zur regionalen Geoidbestimmung mit „dm“ Genauigkeit. Wiss. Arb. Univ. Hannover, Nr. 157.

Brennecke, J., D. Lelgemann, W. Torge, H.-G. Wenzel (1983). A European astro-gravimetric geoid. Deutsche Geod. Komm., Reihe B, Nr. 269, Frankfurt/Main.

Denker, H. (1988). Hochauflösende regionale Schwerefeldbestimmung mit gravimetrischen und topographischen Daten. Wiss. Arb. Univ. Hannover, Nr. 156.

Forsberg, R., C.C. Tscherning (1981). The use of height data in gravity field approximation. J. Geophys. Res. 86, 7843-7854.

Haagmans, R., E. de Min, M.v. Gelderen (1993). Fast evaluation of convolution integrals on the sphere using 1D FFT, and a comparison with existing methods for Stokes' integral. manuscripta geodaetica 18, 227-241.

Moritz, H. (1976). Integral formulas and collocation. manuscripta geodaetica 1, 1-40.

Rapp, R.H., Y.M. Wang, N.K. Pavlis (1991). The Ohio State 1991 geopotential and sea surface topography models. Dept. Geod. Sci. and Surv., Rep. 410, Columbus.

Rapp, R.H., N. Balasubramania (1992). A conceptual formulation of a world height system. Dept. Geod. Sci. and Surv., Rep. 421, Columbus.

Sjöberg, L. (1981). Least squares combination of satellite and terrestrial data in physical geodesy. Ann. Géophys. 37, 25-30.

Torge, W. (1991). Geodesy, 2nd edition. Walter de Gruyter, Berlin, New York.

Torge, W., G. Weber, H.-G. Wenzel (1982). Computation of a high resolution European gravimetric geoid. Proc. 2nd Int. Symp. on the Geoid in Europe and Mediterranean Area, 437-460, Rome.

Weber, G. (1984). Hochauflösende Freiluftanomalien und gravimetrische Lotabweichungen für Europa. Wiss. Arb. Univ. Hannover, Nr. 135.

Wenzel, H.-G. (1982). Geoid computation by least squares spectral combination using integral kernels. Proceed. IAG General Meet., 438-453, Tokyo.