# Investigation of Different Methods for the Combination of Gravity and GPS/Levelling Data †

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**Abstract.** Two different methods for the combined computation of the quasigeoid are compared in a test area in Germany. Both methods are based on the remove-restore technique and use the global geopotential model EGM96, point gravity data with a spacing of a few km, a digital terrain model and GPS/levelling control points (with a spacing of about 25 km).

In method I the global model is combined first with the gravity and terrain data using the least squares spectral combination technique with integral formulas. The resulting height anomalies are given in a 1.0´x 1.5´grid. Then a smooth corrector surface is developed from the GPS/levelling data by least squares collocation, using a signal and a trend component.

The second method (II) is based on a common adjustment of the EGM96 reduced gravity and height anomaly observations using point masses and appropriate weight relations. The point masses are arranged at a depth of 10 km, 30 km and 200 km, and in hilly areas also at a depth of 5 km.

Both techniques are compared from the methodological and numerical point of view. The results are discussed and show an agreement at the cm level.

## 1 Introduction

The Bundesamt für Kartographie und Geodäsie (BKG) and the State Survey Offices have organized the observation of precise GPS/levelling control points in Germany, following a BKG proposal from 1992. The average spacing of the control points is about 25 km. The ellipsoidal GPS heights are referring to the ETRS89 reference system, while the levelled heights are given as normal heights in the DHHN92 system. The main objective of these GPS/levelling control points is to serve for the computation of a new quasigeoid model for Germany, in connection with high resolution gravity and terrain data.

The BKG and the Institut für Erdmessung (IfE) agreed to cooperate on the computation of this new combined quasigeoid, which shall become a standard for the transformation of heights between the ETRS89 and the DHHN92 height system. The present paper describes some first results based on two different combination procedures for a test area in East Germany, as at present all required data sets are only available for this subarea (with a size of about 100,000 km$^2$).

## 2 Data Description

For the quasigeoid determination 4 groups of data are available in the test area (50°N - 55°N, 9°E - 16°E):

- height anomalies from GPS and levelling ( $\zeta_{GPS}$ ),
- terrestrial gravity anomalies ( $\Delta g$ ),
- digital terrain models (*DTM*),
- geopotential models (*GPM*).

Height anomalies from GPS and levelling are available for a total of 196 points (see Fig. 1). The GPS observations were always made in two sessions of 24 hours. The average point-to-point distance is about 25 km. The computation of the GPS heights, $h_{ETRS}$, is based on the European Terrestrial Reference System 1989 (ETRS89). Normal heights, $H^N_{DHHN}$, referring to the DHHN92 height system, were determined for all GPS stations by precise levelling. Considering the accuracy of the levelling heights and of the ellipsoidal heights from GPS, the accuracy of the GPS/levelling quasigeoid heights, $\zeta_{GPS} = h_{ETRS} - H^N_{DHHN}$, is estimated as ± 0.015 m.

Furthermore, for the test area there are more than 70,000 point gravity values available. Outside of the test area there are additional point and mean gravity anomalies available from the IfE and BKG data base. Fig. 2 displays the locations of the gravity data from the IfE data base.

Finally, a high resolution digital terrain model with an original block size of about 30 m as well as the geopotential model EGM96 (Lemoine et al. 1996) were utilized.

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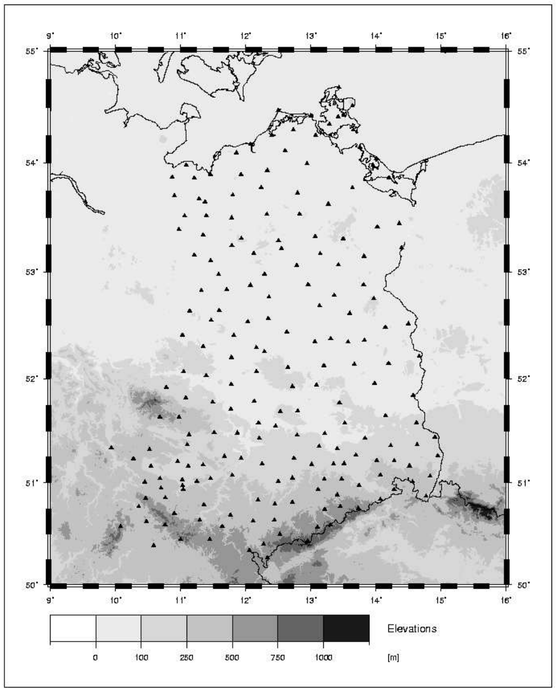


Fig. 1 Locations of GPS/levelling stations and digital terrain model

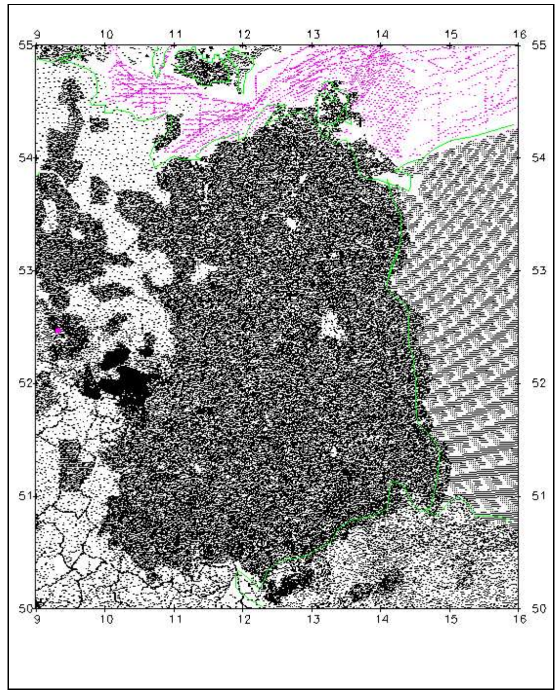


Fig. 2 Locations of gravity stations

## 3 Combination Method I

In 1997, the high resolution European gravimetric (quasi)geoid model EGG97 was computed at the Institut für Erdmessung (IfE), University of Hannover, Germany, operating as the computing centre of the International Association of Geodesy (IAG) Subcommission for the Geoid in Europe (Denker and Torge 1997). The EGG97 model was computed in a 1.0´ x 1.5´ grid and combines about 2.7 million terrestrial gravity data, 700 million terrain data and the spherical harmonic model EGM96 from NASA/NIMA (Lemoine et al. 1996).

The mathematical modelling is based on the spectral combination technique in connection with a remove-restore procedure. Formal error estimates of the resulting height anomalies were derived on the basis of corresponding degree variances. Based on a $\pm 1$ mgal correlated noise for the gravity data, the standard deviations of the height anomalies are $\pm 0.064$ m, while the standard deviations for height anomaly differences are $\pm 0.039$ m over 100 km and $\pm 0.076$ m over 1000 km distance, respectively. The analysis also shows that the major error contribution is coming from the spectral band below degree *l*=360, suggesting that the EGG97 error is predominantly long-wavelength (Denker 1998). This finding is also confirmed by intercomparisons with GPS/levelling data, showing long to medium wavelength discrepancies (see *Denker* 1998, *Milbert* 1995).

This circumstance opens the possibility to develop an empirical corrector surface which relates the given gravimetric quasigeoid model to the reference system of GPS and levelling heights (*Milbert* 1995, *Denker* 1998). It must be understood that such a corrector surface will incorporate systematic errors from ellipsoidal, levelling, and geoidal sources. Modelling of the corrector surface begins by forming residuals in the sense of

$$\zeta_{GPS} - \zeta_{EGG97} = \left(h_{GPS} - H^N\right) - \zeta_{EGG97} = l = t + s + n, \quad (1)$$

where $\zeta_{GPS}$ is the GPS/levelling quasigeoid undulation, computed as the difference of the ellipsoidal height from GPS, $h_{GPS}$, and the normal height from levelling, $H^N$, $\zeta_{EGG97}$ is the quasigeoid undulation from the gravimetric model EGG97, and $l$ are the raw residuals, which are considered as a trend (*t*), signal (*s*), and noise (*n*) component in a least-squares collocation model.

The trend component (*t*) is modelled by a 3-parameter datum shift in the form

$$t = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z, \quad (2)$$

with the ellipsoidal latitude and longitude $\varphi$ and $\lambda$, and the datum shift constants $\Delta X, \Delta Y, \Delta Z$. Instead of $\Delta X, \Delta Y, \Delta Z$ one can also introduce changes in the ellipsoidal coordinates of an initial point, which can be interpreted as a height bias and tilts in NS and WE direction. For the present test area the magnitude of the tilt is 0.18 ppm at an azimuth of 175°. After computing the trend parameters, an empirical covariance function of the detrended residuals (observations), *l*-*t*, was computed and modelled by a simple mathematical function (see Fig. 3).

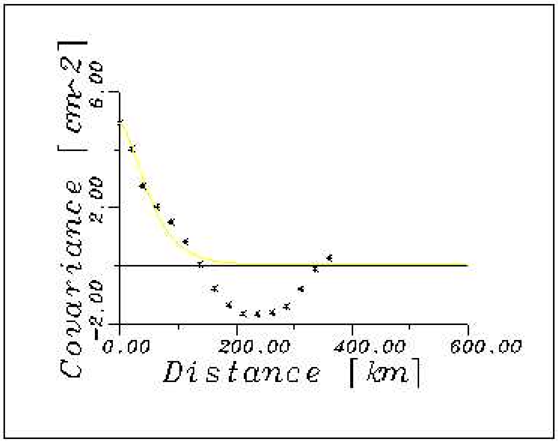


Fig. 3 Empirical Covariance Function (Signal)

We used a second order Markov covariance model in the form

$$Cov(s) = C_o(1 + s/\alpha)\exp(-s/\alpha), \tag{3}$$

where $s$ is the distance, $C_o$ is the signal variance, and $\alpha$ is a parameter that describes the characteristic length of the covariance function. After fixing the signal and error covariance models ($\pm 0.022$ m signal standard deviation, 50 km signal correlation length, $\pm 0.015$ m uncorrelated noise), the signal component can be computed in an arbitrary station P by the formula

$$\hat{s} = \boldsymbol{c}_{\boldsymbol{P}}^{\boldsymbol{T}}(\boldsymbol{C} + \boldsymbol{D})^{-1}(\boldsymbol{l} - \boldsymbol{t})\,. \tag{4}$$

In Eq. (4) $\hat{s}$ is the predicted signal in station P, $\boldsymbol{C}$ is a matrix containing the signal covariances between the observations, $\boldsymbol{D}$ is the noise covariance matrix, and the vector $\boldsymbol{c}_{\boldsymbol{P}}$ contains the signal covariances between the

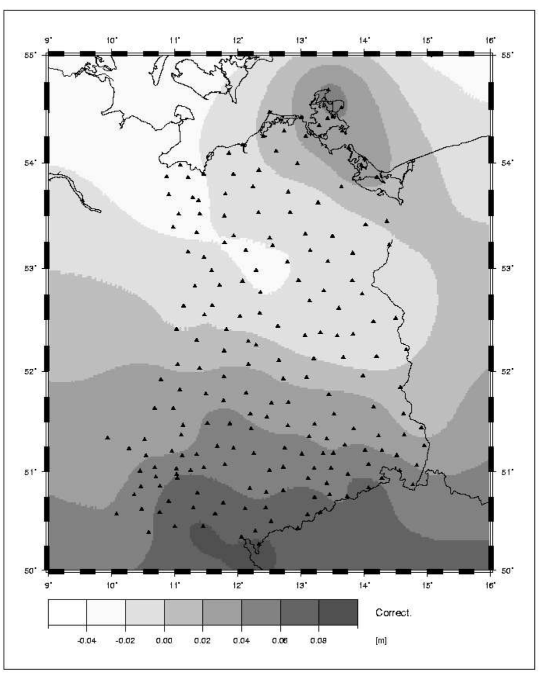


Fig. 4 EGG97 Corrector Surface (Method I)

**Table 1** Statistics of the Individual Model Components for Method I (Computed in 196 GPS/Lev. points)

| Parameter | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|
| $\zeta_{\mathrm{GPS}} - \zeta_{\mathrm{EGG97}}$ | 0.023 | 0.033 | -0.044 | +0.088 |
| $\zeta_{\mathrm{GPS}} - (\zeta_{\mathrm{EGG97}} + t)$ | 0.000 | 0.022 | -0.046 | +0.098 |
| $\zeta_{\mathrm{GPS}} - (\zeta_{\mathrm{EGG97}} + t + s)$ | 0.000 | 0.006 | -0.022 | +0.027 |
| Trend $t$ | 0.023 | 0.024 | -0.027 | +0.060 |
| Signal $s$ | 0.000 | 0.020 | -0.033 | +0.071 |
| Corr. Surf. ($t$+$s$) | 0.023 | 0.031 | -0.033 | +0.082 |

predicted signal and the observations. Finally, the predicted signal and the trend component are added to the original gravimetric quasigeoid (EGG97), yielding the corrected (improved) geoidal surface (denoted as EGG97C) in the form

$$\zeta_{EGG97}^{corr} = \zeta_{EGG97} + t + \hat{s}. \tag{5}$$

The corrector surface, i.e. $t$+$s$, is shown in Fig. 4. The above described technique to combine a gravimetric geoid/quasigeoid with GPS/levelling can be regarded as a stepwise solution, similar to stepwise collocation, where in the first step the gravity and terrain data are combined with the global model, while in the second step the GPS/levelling data are added on the basis of empirical covariance modelling.

The statistics of all relevant model components were computed in the 196 GPS/levelling stations and are presented in Table 1. The raw residuals $\zeta_{GPS} - \zeta_{EGG97}$ according to Eq. (1) show a mean value of 0.023 m and a standard deviation of $\pm 0.033$ m. The detrended residuals $(\zeta_{GPS} - \zeta_{EGG97} - t)$ show a standard deviation of $\pm 0.022$ m with maximum values of 0.098 m. The larg-

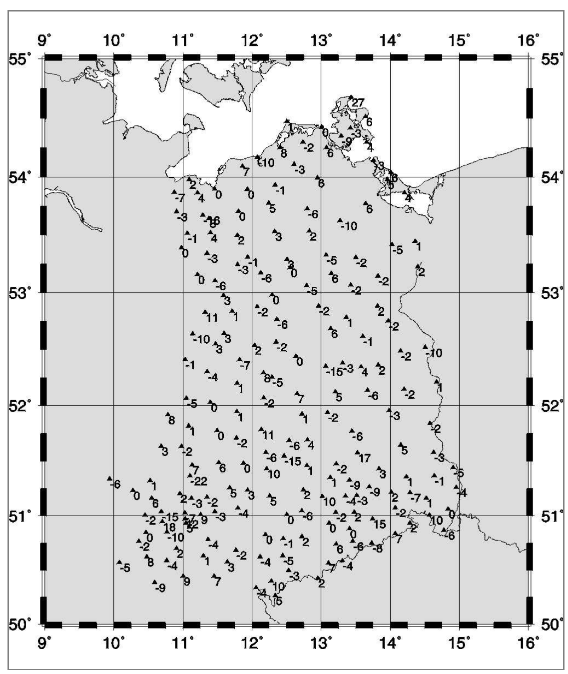

Fig. 5 Residuals for Method I (in mm) in the 196 GPS/levelling stations

est values are located in the north around the island of Rügen. The predicted signal has a standard deviation of ± 0.020 m with maximum values up to 0.071 m. The largest values are again found in the north. Further interpretation of the signal component is difficult, because it contains effects from GPS, levelling and the gravimetric quasigeoid. The residuals about the predictions, i.e. $\zeta_{GPS} - \zeta_{EGG97} - t - s$ , are shown in Fig. 5. The Rms value was found to be ± 0.006 m, being significantly smaller than the assigned data noise. The remaining maximum discrepancies are only ± 0.027 m. This documents the efficiency of the procedure.

## 4 Combination method II

In this combination method, the GPS/levelling quasigeoid heights and the gravity anomalies are introduced as observations in an adjustment of point masses. The adjustment procedure is also based on a remove-restore technique, where the observations are reduced for the long-wavelength effect of the geopotential model EGM96 ($\zeta_{GPM}$, $\Delta g_{GPM}$). Previous investigations showed that short-wavelength effects from a digital terrain model were not giving a significant improvement for the current setup of the point mass adjustment. Thus terrain effects were neglected in the point mass modelling to date.

The residual gravity anomalies $\Delta g'$ and height anomalies $\zeta'$, that are used in the point mass adjustment, are thus defined as :

$$\Delta g' = \Delta g - \Delta g_{GPM} \, , \tag{6}$$

$$\zeta' = \zeta_{GPS} - \zeta_{GPM} \, . \tag{7}$$

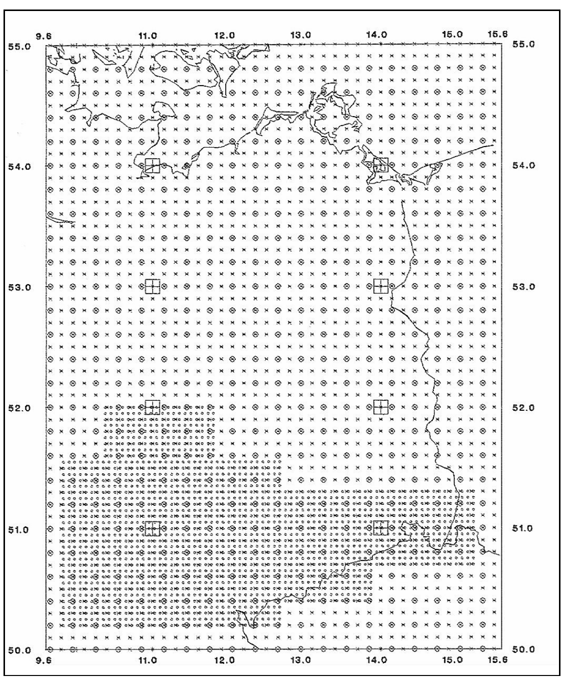


Fig. 6 Locations of point masses (Method II)

The relations between the masses $m$ and the residual height and gravity anomalies (observation equations) are as follows:

$$\Delta g' = G \sum \frac{m(H - H')}{d^3} \, , \tag{8}$$

$$\zeta' = \frac{G}{\gamma_0} \sum \frac{m}{d} \, . \tag{9}$$

In these equations $G$ is the Newton's gravitational constant and $d$ is the distance.

The accuracy of both observation types is considered in a weight matrix. As a priori accuracies, ± 0.015 m are introduced for the height anomalies and ± 1 mgal for the gravity anomalies. Here it should be noted that in the adjustment the gravity anomalies are introduced as mean values with a block size of 2 km and 5 km. The total number of mean values is 25700.

Investigations showed that a hierarchical arrangement of the point masses at different depths leads to optimal results (ratio of observations, unknowns and accuracy). The point masses, arranged at a depth of 10 km and with a distance of 0.1°x 0.15° shall approximate mainly the short-wavelength parts of the quasigeoid. The point masses at a depth of 30 km with a distance of 0.2°x 0.3° cover basically those frequencies which are determined by the height anomalies from GPS and levelling. 8 point masses at a depth of 200 km shall compensate the long wavelength and slope influences. In hilly areas additional point masses with a distance of 0.05° x 0.075° are arranged at a depth of 5 km (Fig. 6).

The quasigeoid model from this computation is denoted as BKG98.

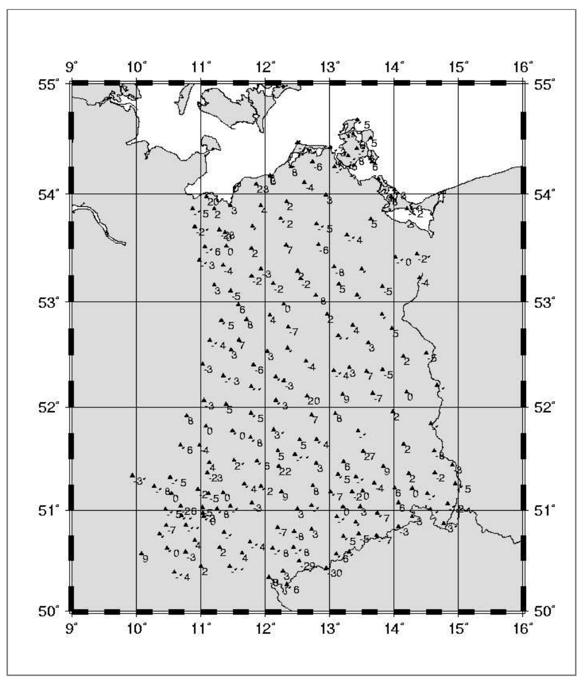


Fig. 7 Residuals for Method II (in mm) in the 196 GPS/levelling stations

The Rms residual of the GPS/levelling derived height anomalies is ± 0.011 m. For the northern flat country area the Rms value is ± 0.010 m, while the value for the southern hilly area is ± 0.012 m, respectively. The individual residuals are also shown in Fig. 7. The Rms residual of the gravity anomaly is ± 2.2 mgal.

## 5 Comparison of both methods and discussion

The combined quasigeoid solutions from method I (EGG97C) and method II (BKG98) were intercompared in the area covered by the GPS/levelling stations. The differences between EGG97C and BKG98 are displayed in Fig. 8. The Rms difference is ± 0.010 m. The maximum differences are located close to the boundary of the comparison area (see Fig. 8), the main reason being that the BKG98 solution does not include detailed gravity data outside the area covered by the GPS/levelling stations. Therefore, the comparison was repeated with a 10 km border area being excluded. For this case the Rms difference is ± 0.009 m with maximum values of 0.046 m. In the inner area the maximum differences of about 0.045 m are found around 52°N and 12°W. Especially in this area, the differences show structures that are correlated with the location of the point masses. It should also be noted that in this region the dense point mass grid (5 km) ends (see Fig. 6). In the future this phenomena needs to be studied in more detail. Furthermore, the differences were also analysed along profiles and spectra were computed. The spectra show peaks at half wavelengths of about 10 km and 20 km, which corresponds to the grid spacing used in the point mass modelling.

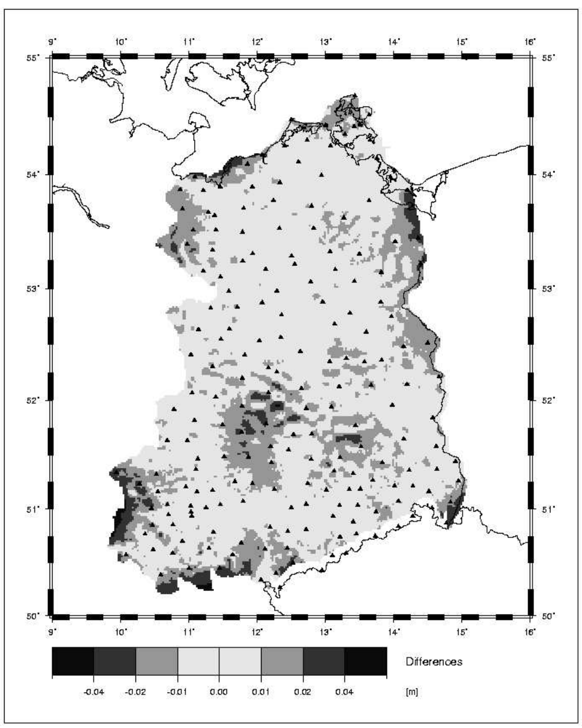


Fig. 8 Height Anomaly Differences EGG97C - BKG98

In another analysis the residuals from both methods (see Fig. 5 and 7) were studied. The correlation between the residuals from both methods is about 65 %. This shows that both methods have the same tendency in the GPS/levelling stations, and larger residuals may also indicate small height errors in these stations.

A strong test of the used combination procedures would be to intercompare the two combined quasigeoid solutions with an independent GPS/levelling data set. However, as at present such a data set is not available, additional solutions were derived using only about one half of the GPS/levelling control points in the computation of the combined quasigeoid models, while the remaining stations were used for a comparison only. This was done for both combination procedures, and the corresponding solutions, based on only 94 GPS/levelling points, are denoted as EGG97C (B) and BKG98 (B), respectively. The Rms residuals for the previously described solutions and the corresponding B solutions are given in Table 2. For the B solutions, the statistics are given for the 94 stations used in the development of the corresponding solutions as well as for the remaining 102 stations used only for the evaluation. The table shows a slight increase of the residuals in the independent comparison stations, but the results are still very satisfactory.

**Table 2** Statistics of the residuals in the GPS/levelling points

| Solution | | Mean | Std. Dev. | Min. | Max |
|---|---|---|---|---|---|
| BKG98 | (196) | 0.000 | 0.011 | -0.031 | +0.031 |
| EGG97C | (196) | 0.000 | 0.006 | -0.022 | +0.027 |
| BKG98 (B) | (94) | 0.001 | 0.013 | -0.040 | +0.039 |
| | (102) | 0.001 | 0.015 | -0.033 | +0.034 |
| EGG97C (B) | (94) | 0.000 | 0.005 | -0.013 | +0.025 |
| | (102) | 0.001 | 0.011 | -0.033 | +0.024 |

## 6 Conclusions

The two investigated methods to combine GPS/levelling data with gravimetric data are based on a totally different concept. Both methods show a satisfactory agreement of ± 0.01 m (Rms). The maximum differences of about 0.045 m are located in the inner area and show a correlation with the location of the point masses, indicating that the point mass modelling should be improved. The residuals in the GPS/levelling control points from both combination procedures show a high correlation (65 %), indicating that small height errors (resulting from inaccurate centering data, different observation epochs of the GPS and levelling data, etc.) might still exist. Before doing the final computations for the entire area of Germany, the existing problems have to be further studied and clarified.

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