Evaluation and Improvement of the EGG97 Quasigeoid Model for Europe by GPS and Leveling Data †

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Abstract. A high resolution quasigeoid model, EGG97, was computed in 1997 at the Institut für Erdmessung (IfE), operating as the computing center of the International Association of Geodesy (IAG) Subcommission for the Geoid in Europe. The EGG97 model was computed in a 1.0'×1.5' grid and combines about 2.7 million terrestrial gravity data, 700 million terrain data and the spherical harmonic model EGM96 from NASA/NIMA. The mathematical modeling is based on the spectral combination technique in connection with a remove-restore procedure. Formal error estimates of the resulting height anomalies were derived on the basis of corresponding error degree variances.

The EGG97 quasigeoid model is evaluated by intercomparisons with different GPS and leveling data sets. Most of the comparisons show a good agreement in the order of ±1 cm over short wavelengths (few 100 km), while systematic discrepancies in the order of 0.1 ppm to 1 ppm usually exist over medium to long wavelengths (several 100 km to 1000 km). This indicates medium to long wavelength errors in the employed global geopotential model and the terrestrial gravity data. The discrepancies between the GPS, the leveling and the EGG97 data are modeled by a trend and a signal component. For the trend component a 3-parameter datum shift model is used, which can be interpreted as a tilted plane model with a height bias and tilts in NS and WE direction. The detrended differences are further investigated by computing an empirical covariance function. A simple analytical covariance model with an appropriate characteristic length is used to approximate the empirical covariance function. A least-squares collocation predictor then leads to the development of a smooth corrector surface, including the trend and the signal component. This corrector surface can be added to the EGG97 quasigeoid model, relating directly the corresponding GPS and leveling datums. The procedure is tested using a GPS/leveling data set for France.

1. Introduction

Today the Global Positioning System (GPS) and other space techniques provide ellipsoidal heights at unprecedented accuracies in the order of ±1 cm to a few cm at regional to global scales. On the other hand, many applications in geodesy, geophysics and engineering require physically defined heights related to the Earth's gravity field (orthometric or normal heights), typically produced by geometric leveling. Therefore, for the conversion and combination of these fundamentally different height systems, the geoid/quasigeoid must be known with an accuracy comparable to the accuracy of GPS and leveling.

In order to promote the development of an improved geoid/quasigeoid model for Europe, the International Association of Geodesy (IAG) established a Geoid Subcommission for Europe in 1990, and the Institut für Erdmessung (IfE) was asked to serve as the computing center in this project. IfE has produced several geoid/quasigeoid models since 1990, combining high resolution gravity and terrain data with a global geopotential model. The latest version of these models is called EGG97 and uses the global model EGM96 from NASA/NIMA.

The paper summarizes the development of the EGG97 model, followed by an evaluation with GPS/leveling data. For the combination of the EGG97 model with GPS/leveling data a least-squares collocation approach with a trend and a signal component is investigated.

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2. The Gravimetric Geoid/Quasigeoid Model EGG97

The IfE gravity field modeling effort for Europe has concentrated on the calculation of height anomalies or quasigeoid undulations ζ. This has the advantage that only gravity field data observed at the Earth's surface and in its exterior enter into the calculations, while no assumptions about the gravity field in the Earth's interior are needed. Subsequently, a transformation from height anomalies ζ to geoid undulations N is performed by introducing a density model.

The remove-restore technique was used to combine the global geopotential model EGM96 (ℓ_{max}=360) from NASA/NIMA (Lemoine et al. 1997) with high resolution digital terrain and gravity data. For the field transformation from gravity to height anomalies, the least-squares spectral combination technique (see e.g. Wenzel 1982) was applied in order to reduce long wavelength distortions, which may result from the use of Stokes's formula (see e.g. Denker et al. 1995 and 1997). The spectral weights needed to compute the modified integral kernel were computed from the error degree variances of EGM96 and the terrestrial gravity data, where the latter ones were computed from an error covariance function using correlated noise (for details see Denker and Torge 1997). It was decided to do the combination only up to degree 50, while above degree 50 the total weight was given to the terrestial gravity data. However, this does not imply that the global model is completely disregarded above degree 50, as especially in areas with larger data gaps the high degree gravity information of the model is considered in the gridding process and thus practically taken over in the final quasigeoid model. The numerical evaluation of the integral formula was done by a 1D FFT technique in connection with a detailed/coarse grid approach to further speed up the computations.

In the course of the European Geoid Project, about 2.7 million gravity data and about 700 million topographical height data were included in the project data base. While land gravity data with a resolution of at least 10 km were attained for all European countries, the marine gravity data coverage is still insufficient for some regions (see e.g. Torge and Denker 1998). For this reason, the ship data were merged with ERS-1 gravity anomalies (Andersen et al. 1996) in most parts of the European seas.

The digital terrain models (DTMs) were regridded to a common block size of 7.5"×7.5", with existing gaps being filled by values from ETOPO5. The DTMs were employed for the modeling of the short wavelength gravity field components using the residual terrain model (RTM) reduction technique, where the reference topography was constructed from the DTMs using a 15'×15' moving average filter.

The geoid/quasigeoid was computed in a 1.0'×1.5' grid covering the area from 25° N - 77° N and 35° W - 67.4° E. This yields 3,120 × 4,096 = 12,779,520 grid points. The major contribution to the final quasigeoid EGG97 comes from the spherical harmonic model EGM96 with values ranging from -43.3 m to +67.9 m and a standard deviation of ±25.6 m. The standard deviations of the contributions from the DTM and the terrestrial gravity data are ±0.03 m and ±0.41 m, respectively. However, the maximum DTM effects are about 0.8 m, while the maximum effects of the terrestrial gravity data are 4.3 m. Furthermore, for the final EGG97 model, a zero-degree undulation correction of -0.5 m and a tidal correction (to refer the undulations to the zero tide system recommended by IAG) were applied. The Molodensky correction terms, which can attain maximum values of 10 cm in the Alps and 1 cm in the highlands, were neglected so far. Finally, a conversion to the geoid was done by introducing a Bouguer plate model with constant density, corresponding to the so-called Helmert heights. The EGG97 geoid and quasigeoid model are both available on CD-ROM as announced in the Journal of Geodesy (1997). For more details on the development of EGG97 see Denker and Torge (1997).

The spectral combination technique also permitted the derivation of error estimates for the resulting height anomalies on the basis of corresponding error degree variances (for details see e.g. Wenzel 1982). Figure 1 shows the square root of the height anomaly error degree variances based on gravity data only (error variance is $(2\ mgal)^2$, correlated noise, details see Denker and Torge 1997), the global model EGM96 as well as the combined European quasigeoid model EGG97. From Fig. 1 it is evident that the long wavelength gravity field information from EGM96 is superior to the terrestrial gravity data, while medium wavelength components can be determined more accurately from the collected gravity data. Furthermore, standard deviations of the EGG97 height anomalies and height anomaly differences were computed

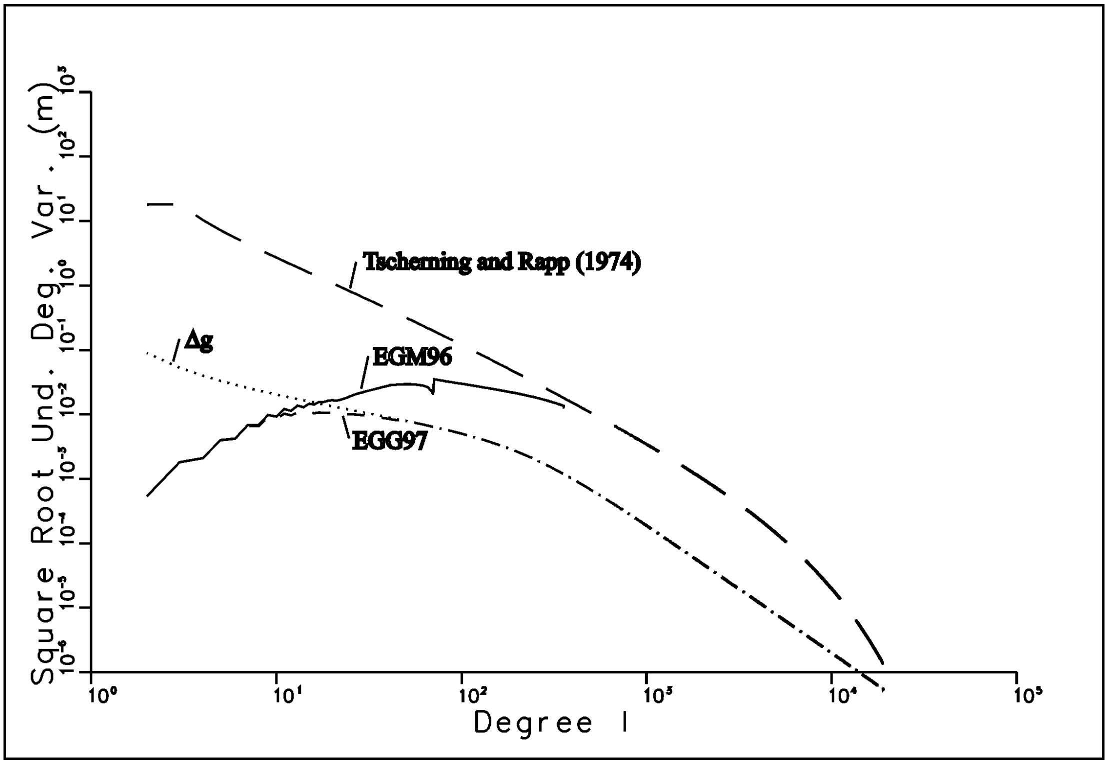

Fig. 1. Square root of height anomaly error degree variances based on gravity data Δg, the EGM96 global model, the EGG97 combined quasigeoid, and the Tscherning and Rapp (1974) model.

Table 1. Standard deviations of height anomalies ζ and height anomaly differences Δζ derived from the EGG97 model (standard deviation of Δg is 2 mgal; units are meters).

Degree l	ζ	Δζ Distance		
		10 km	100 km	1000 km
2 - 20	0.0370	0.0006	0.0062	0.0500
21 - 50	0.0499	0.0020	0.0195	0.0727
51 - 90	0.0413	0.0032	0.0306	0.0592
91 - 180	0.0386	0.0055	0.0478	0.0550
181 - 360	0.0262	0.0072	0.0423	0.0372
361 - 1000	0.0138	0.0079	0.0183	0.0195
1001 - 2000	0.0033	0.0041	0.0047	0.0046
2001 - 10000	0.0013	0.0020	0.0018	0.0018
10001 - ∞	0.0001	0.0002	0.0002	0.0002
Total	0.0892	0.0134	0.0761	0.1269

from the corresponding error degree variances. The results are shown in Table 1. The standard deviations are provided for the complete spectrum as well as for some selected spectral bands. The standard deviations of the height anomalies are ±8.9 cm, while the standard deviations for height anomaly differences are ±7.6 cm over 100 km and ±12.7 cm over 1000 km distance, respectively. In case of a more optimistic error estimate for the terrestrial gravity data of ±1 mgal we obtain standard deviations for the height anomalies of ±6.4 cm, while for height anomaly differences we get values of ±3.9 cm over 100 km and ±7.6 cm over 1000 km distance. Table 1 also shows that the major error contribution is coming from the spectral band below degree l=360. This suggests that the error in high resolution geoid/quasigeoid models is predominantly long-wavelength. Moreover, this documents that an improvement of the long to medium wavelength gravity field components by a dedicated space gravity field mission as well as accurate terrestrial gravity data is needed.

3. Evaluation of the EGG97 Quasigeoid Model by GPS and Leveling Data

For the evaluation of the EGG97 quasigeoid model a number of GPS/leveling data sets were collected. However, in this paper we will restrict the quality assessment of EGG97 to the comparison with three GPS/leveling data sets covering small (NDS92, Lower Saxony, 300 km extension), medium (RBF, France, 1000 km extension) and large scales (European GPS traverse from Austria to northern Norway, 3000 km length). For these three campaigns we have precise ellipsoidal heights and rigorously computed normal heights available. The statistics of the discrepancies between the GPS/leveling data and some recent European and global quasigeoid solutions are given in Table 2. The comparisons were always done using a bias fit as well as a bias and tilt fit in order to account for inaccuracies in the absolute positioning and for long wavelength errors of all data sets involved (GPS, leveling, quasigeoid).

From Table 2 it becomes clear that the new quasigeoid solution EGG97 is a significant improvement over the older models EGG1 (Torge et al. 1982) and EAGG1 (Brennecke et al. 1983) as well as the global model EGM96 (Lemoine et al. 1997), with the RMS discrepancies decreasing by a factor of 2 to 5. In the EGG97 comparisons one can observe for all three campaigns a significant improvement for the bias and tilt fit versus the bias fit, thus indicating that small long wavelength discrepancies exist between the gravimetric quasigeoid and the GPS/leveling data in the respective regions (magnitude 0.1 to 0.5 ppm).

For the local GPS/leveling data set NDS92 a RMS discrepancy of ±0.038 m was found for the bias fit, while the corresponding value for the bias and tilt fit is only ±0.013 m, being at the noise level of the data. For the French and the European data set the RMS differences for the bias fit are ±0.128 m and

Table 2. Statistics of the differences from the comparison of selected quasigeoid solutions with different GPS/leveling data sets (units are meters).

Quasigeoid Solution	Bias Fit		Bias + Tilt Fit	
	RMS	Max.	RMS	Max.
NDS92 (Lower Saxony; 41 stations)				
EGG1 (1982)	0.108	0.368	0.061	0.221
EAGG1 (1983)	0.069	0.171	0.063	0.162
EGM96 (1996)	0.148	0.462	0.136	0.567
EGG97 (1997)	0.038	0.090	0.013	0.033
RBF (France; 965 stations)				
EGG1 (1982)	0.664	2.937	0.381	2.079
EAGG1 (1983)	0.460	1.882	0.387	1.803
EGM96 (1996)	0.369	2.025	0.301	1.698
EGG97 (1997)	0.128	0.353	0.080	0.484
European GPS Traverse (67 stations)				
EGG1 (1982)	0.605	1.351	0.275	0.778
EAGG1 (1983)	0.240	0.607	0.175	0.501
EGM96 (1996)	0.304	0.875	0.251	1.114
EGG97 (1997)	0.294	0.793	0.175	0.470

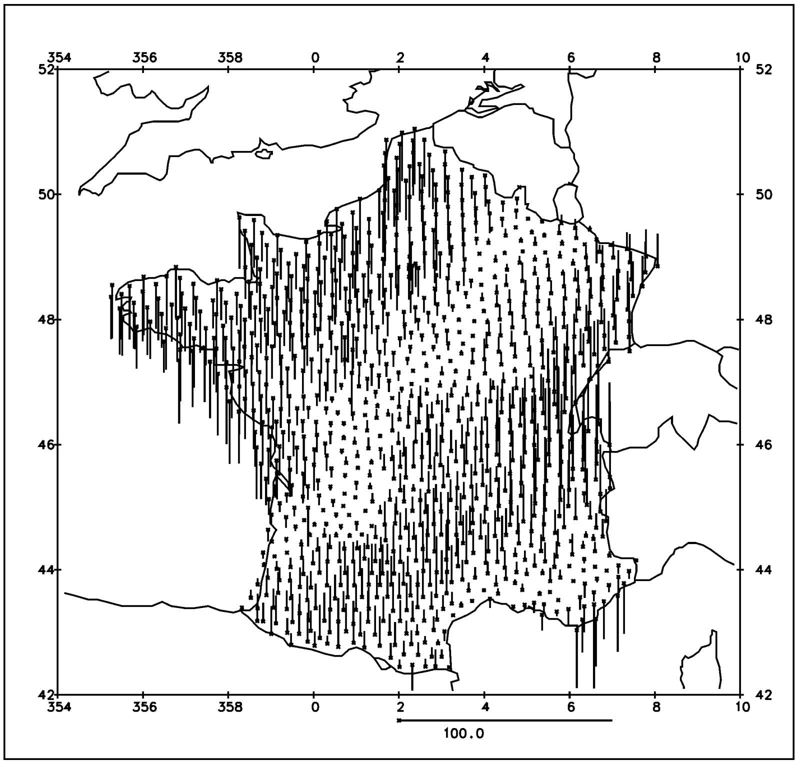

Fig. 2. ζ_{GPS}-ζ_{EGG97} after bias reduction (units are cm).

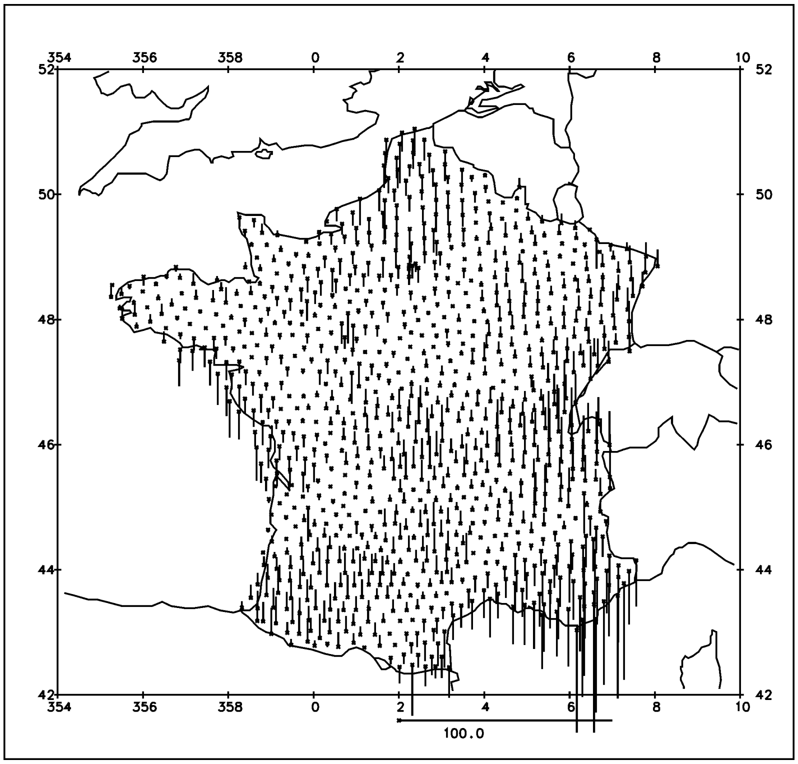

Fig. 3. ζ_{GPS}-ζ_{EGG97} after bias and tilt reduction (units are cm).

±0.294 m, which reduce to ±0.080 m and ±0.175 m for the bias and tilt fit, respectively. Figure 2 shows the discrepancies between the GPS/leveling data from the RBF campaign and the EGG97 quasigeoid after the bias fit, while the corresponding results for the bias and tilt fit are displayed in Fig. 3. From Fig. 2 and 3 and from corresponding difference plots of other campaigns, one can observe, in general, a very good short wavelength agreement in the order of ±0.01 m, while medium to long wavelength discrepancies exist in most cases. As the magnitude of these discrepancies is significantly larger than the error budgets of present-day GPS results and leveling, it is believed that the main reason for the existing discrepancies are medium to long wavelength errors in the quasigeoid, coming from corresponding errors in the employed global model and the terrestrial gravity data.

4. Combination of the EGG97 Quasigeoid Model with GPS and Leveling Data

The experiences with intercomparisons of GPS/leveling against high resolution geoid and quasigeoid models have shown long to medium wavelength discrepancies (see also Milbert 1995). This circumstance opens the possibility to develop an empirical corrector surface which relates a given gravimetric geoid/quasigeoid model to the reference system of GPS and leveling heights (Milbert 1995). It must be understood that such a corrector surface will incorporate systematic errors from ellipsoidal, leveling, and geoidal sources. However, if the error contributions from the ellipsoidal GPS heights and the leveling data are of minor importance, then the application of the corrector surface will lead to a corresponding improvement of the geoidal surface. Moreover, such a corrected geoid/quasigeoid surface has the important property, that it directly relates the ellipsoidal GPS heights to the national system of leveling heights, which is a strong desire for practical applications in different fields.

Modeling of the corrector surface begins by forming residuals in the sense of

$$\begin{aligned}\zeta_{GPS} - \zeta_{EGG97} &= (h_{GPS} - H^N) - \zeta_{EGG97} = \\ &= l = t + s + n\,, \end{aligned} \qquad (1)$$

where ζ_{GPS} is the GPS/leveling quasigeoid undulation, computed as the difference of the ellipsoidal height from GPS, h_{GPS}, and the normal

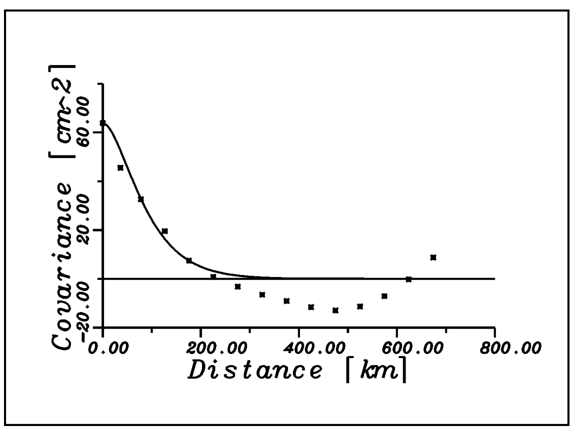

Fig. 4. Empirical and analytical covariance function for the detrended residuals of the RBF data set.

Table 3. Statistics of the corrector model (EGG97/RBF) computed in the 965 GPS/leveling stations (units are meters).

Parameter	Mean	Std.Dev.	Min.	Max.
ζ_{GPS}-ζ_{EGG97}	-0.533	0.128	-0.830	-0.180
ζ_{GPS}-(ζ_{EGG97}+t)	0.000	0.080	-0.484	+0.209
ζ_{GPS}-(ζ_{EGG97}+t+s)	0.000	0.013	-0.064	+0.067
Trend t	-0.533	0.100	-0.783	-0.312
Signal s	0.000	0.077	-0.452	+0.182

height from leveling, H^N, ζ_{EGG97} is the quasigeoid undulation from the gravimetric model EGG97, and l are the raw residuals, which are considered as a trend (t), signal (s), and noise (n) component in a least-squares collocation model. The trend component (t) is modeled by a 3-parameter datum shift in the form

$$t = \cos\varphi\cos\lambda\Delta X + \cos\varphi\sin\lambda\Delta Y + \sin\varphi\Delta Z\,, \qquad (2)$$

with the ellipsoidal latitude and longitude φ and λ, and the datum shift constants $\Delta X, \Delta Y, \Delta Z$. Instead of $\Delta X, \Delta Y, \Delta Z$ one can also introduce changes in the ellipsoidal coordinates of an initial point, which can be interpreted as a height bias and tilts in NS and WE direction. The latter interpretation is commonly used and more descriptive. After computing the trend parameters, an empirical covariance function of the detrended residuals (observations), l-t, can be computed and modeled by a simple mathematical function. In the following we use a second order Markov covariance model in the form

$$Cov(s) = C_0\,(1 + s/\alpha)\exp(-s/\alpha)\,, \qquad (3)$$

where s is the distance, C_0 is the signal variance, and α is a parameter that describes the characteristic length of the covariance function. After fixing the signal and error covariance models, the signal component can be computed in an arbitrary station P by the formula

$$\hat{s} = \mathbf{C}_\mathbf{P}^\mathbf{T}(\mathbf{C}+\mathbf{D})^{-1}(\mathbf{l}-\mathbf{t})\,. \qquad (4)$$

In Eq. (4) $\hat{s}$ is the predicted signal in station P, $\mathbf{C}$ is a matrix containing the signal covariances between the observations, $\mathbf{D}$ is the noise covariance matrix, and the vector $\mathbf{C_P}$ contains the signal covariances between the predicted signal and the observations. Finally, we add the predicted signal and the trend component to the original gravimetric geoid/quasigeoid and obtain the corrected (improved) geoidal surface in the form

$$\zeta^{\mathrm{corr}}_{\mathrm{EGG\,97}} = \zeta_{\mathrm{EGG\,97}} + t + \hat{s}\,. \qquad (5)$$

The above described technique to combine a gravimetric geoid/quasigeoid with GPS/leveling can be regarded as a stepwise solution, similar to stepwise collocation, where in the first step the gravity and terrain data are combined with the global model, while in the second step the GPS/leveling data are added. The major difference between the above procedure and stepwise collocation is that in the above procedure the covariance modeling is only done empirically as compared to the rigorous covariance propagation in stepwise collocation.

In the following we will use the French GPS/leveling data set RBF as an example to test the approach described above. The original data set was kindly provided by IGN, France, and contains 987 stations with 8 stations having an error flag. In the evaluation of EGG97 (see previous section) and in the following investigations we have excluded 14 additional stations, where the detrended (bias and tilt reduced) residuals showed significant disagreement to nearby stations. Hence 965 stations are used in this study. The trend component was computed according to Eq. (2) and is shown in Fig. 5. The magnitude of the computed tilt is about 0.45 ppm at an azimuth of about 127°.

The detrended residuals (see also Fig. 3) were used to compute an empirical covariance function. The result is shown in Fig. 4, together with the analytical covariance function according to Eq. (3). The signal variance was set to $C_0 = (8\,cm)^2$ and the characteristic length was defined as $s_{1/2} = 80\,km$. The noise variance was set to $(2\,cm)^2$ according to the documentation provided with the GPS/leveling data. The signal component predicted according to Eq. (4) is portrayed in Fig. 6.

Statistics of all relevant model components were computed in the 965 GPS/leveling stations and are presented in Table 3. The raw residuals ζ_{GPS}-ζ_{EGG97} according to Eq. (1) show a mean value of -0.533 m and a standard deviation of ±0.128 m. The significant mean value is mainly related to the French vertical datum definition (IGN69) on the basis of the Marseille tide gauge, as compared to the United European Leveling Network (UELN) which is based on the Amsterdam tide gauge. The detrended residuals (ζ_{GPS}-ζ_{EGG97}-t) show a standard deviation of ±0.080 m with maximum values of about 0.5 m. The largest values (see Fig. 3 and 6) are located in southern France close to the Mediterranean Sea, indicating data problems in this region. The predicted signal has a standard deviation of ±0.077 m with maximum values up to about 0.5 m. The largest values (see Fig. 6) are again found in southern France. Fig. 6 also shows some highs and lows in land and ocean areas outside of France which are extrapolations and not reliable. Further interpretation of the signal component is difficult, because it contains effects from GPS, leveling and the quasigeoid. The residual misfit about the predictions of Eq. (4), i.e. ζ_{GPS}-ζ_{EGG97}-t-s, was found to be ±1.3 cm, being in general agreement with the assigned data noise. The

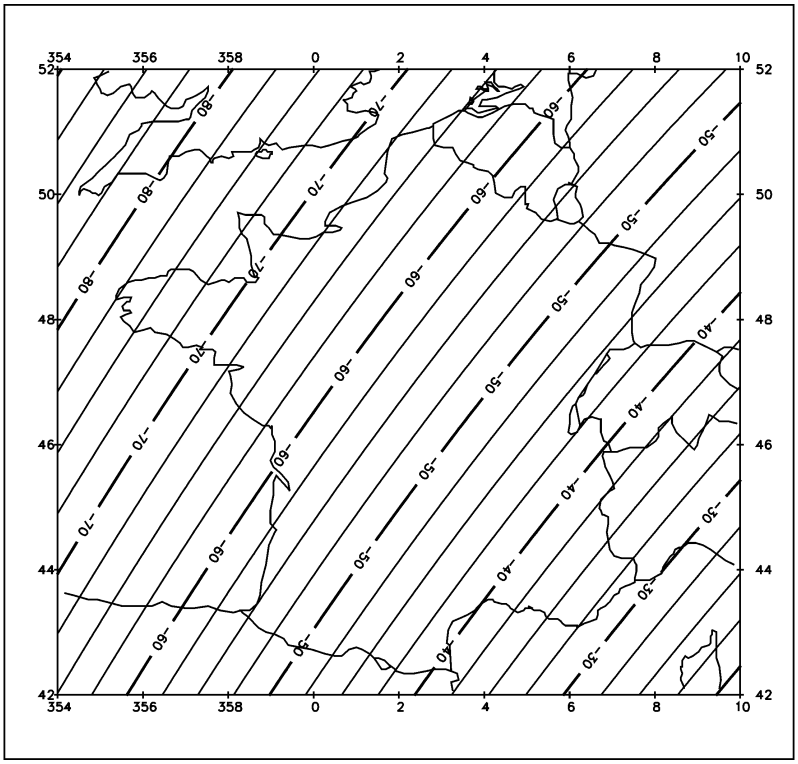

Fig. 5. Trend component (contour interval is 2.5 cm).

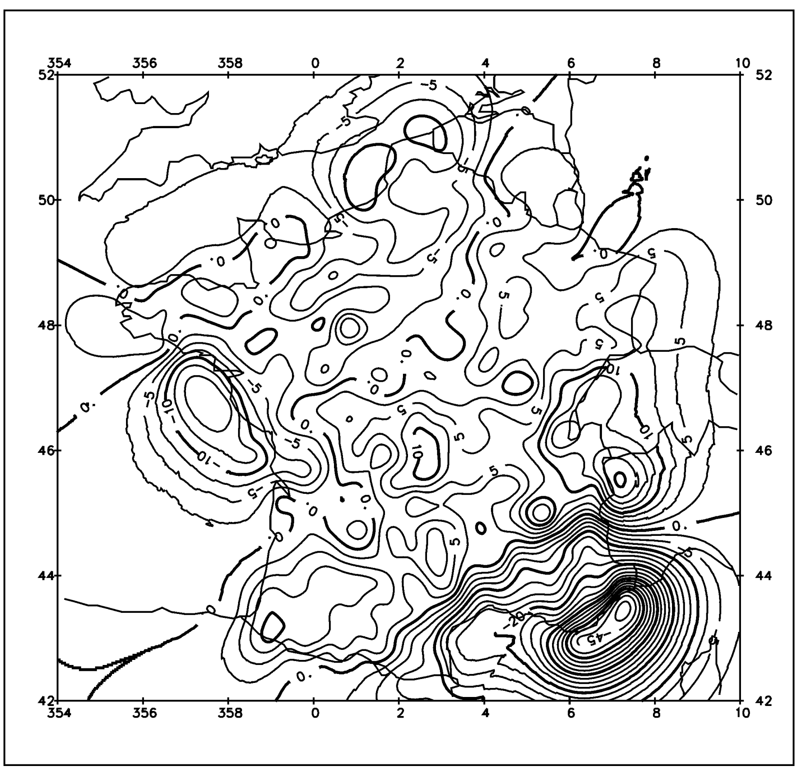

Fig. 6. Signal component (contour interval is 2.5 cm).

remaining maximum discrepancies are only about ±6.5 cm. This documents the efficiency of the procedure.

A strong test of the efficiency of the above procedure to improve an existing geoid/quasigeoid model by adding a corrector surface, including a trend and a signal component, would be to intercompare the corrected geoidal surface with other independent GPS/leveling data. However, at present such a data set is not available. Therefore an investigation was done using only selected GPS/leveling stations from the entire data set to compute the corrector surface, followed by a comparison of the corrected model in the remaining (independent) GPS/leveling stations. The results of this investigation are shown in Table 4. The selection of the stations used for the computation of the corrector surface was done by an auxiliary grid where only the stations closest to the grid knots are retained. The number of stations selected for the computation of the corrector surface is listed in column 2 of Table 4. The remaining stations used for the evaluation of the corrected geoidal surface are listed in column 3 of Table 4. For the sake of completeness, also the extreme cases of using all 965 GPS/leveling stations for the computation of the corrector surface (first line, see also Table 3) and using none of the GPS/leveling stations, i.e. neglecting the signal component (last line, see also Table 3), are included. Table 4 shows that the standard deviations of the residuals are increasing significantly when using only selected GPS/leveling stations instead of the full data set of 965 stations. However, the standard deviations remain below 3.0 cm for all configurations up to a selection distance of about 60-70 km. This shows that even with a significantly thinned (by a factor of 5-7) input data set, the procedure for the computation of the corrector surface is very efficient.

Conclusions

A high resolution geoid and quasigeoid model, EGG97, was computed by combining gravity and terrain data with the global geopotential model EGM96 from NASA/NIMA. The mathematical modeling is based on the spectral combination technique together with a remove-restore procedure. Formal error estimates for the resulting height anomalies and height anomaly differences show that the major error contribution is coming from the spectral band below degree l=360, suggesting that the error in the high resolution geoid/quasigeoid model EGG97 is predominantly long-wavelength.

The practical evaluation of the EGG97 model by GPS/leveling data confirms the existence of long to medium wavelength discrepancies, while the agreement at short wavelengths is usually at the level of ±1 cm. The long and medium wavelength discrepancies are modeled by a tilted plane trend component and a signal component in a least-squares collocation procedure. The method was tested using a GPS/leveling data set for France. A simple empirical covariance function with a characteristic length of 80 km was found to fit the

Table 4. Comparison of ζ_{GPS} and ζ^{corr} for different input station configurations (units are meters).

Select. Grid [km]	Input Stations [#]	Eval. Stations [#]	Mean [m]	Std.Dev. [m]	Min. [m]	Max. [m]
-	965	965	0.000	0.013	-0.064	+0.067
30	619	346	0.000	0.026	-0.114	+0.088
40	372	593	0.001	0.026	-0.104	+0.125
50	249	716	0.002	0.027	-0.115	+0.111
60	181	784	0.000	0.028	-0.130	+0.129
70	138	827	0.001	0.030	-0.200	+0.124
80	107	858	0.000	0.032	-0.124	+0.195
90	87	878	0.004	0.035	-0.130	+0.183
100	73	892	-0.002	0.035	-0.147	+0.178
125	52	913	0.006	0.036	-0.173	+0.168
-	0	965	0.000	0.077	-0.452	+0.182

detrended differences between the GPS/leveling and EGG97 data. The least-squares collocation predictor led to a smooth corrector surface, including the trend and the signal component. The RMS difference between the GPS/leveling and EGG97 data is ±8.0 cm when the tilted plane trend model is considered, and reduces to ±1.3 cm when also the signal component is taken into account. This documents the efficiency of the procedure. Investigations with a thinned GPS/leveling data set showed that even with a spacing of the GPS/leveling control points up to 60-70 km, the corrector surface can be computed at an accuracy level of a few cm.

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