

## Introduction

Global Navigation Satellite Systems (GNSS) are one-way ranging systems. Consequently, receiver and satellites' time scales have to be synchronized. This is generally realized by introducing satellite and receiver clock errors w.r.t. GNSS system time. Corrections for the satellite clock errors are broadcasted by the system provider, e.g. via GNSS navigation message. In contrast, due to the limited long-term frequency stability of the receiver's internal quartz oscillator and its generally poor accuracy, the receiver clock error has to be estimated epoch-by-epoch. This leads to (elevation-dependent) high mathematical correlations of 83–99% between the height component and the clock parameters (Fig. 1). As one consequence, the station height is typically determined about three times worse than the horizontal coordinates.

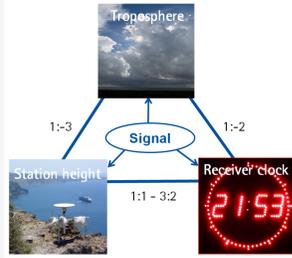


Fig. 1: Relationship between the parameters tropospheric delay, station height, and receiver clock error

This situation can be improved when using more stable clocks and modeling their behavior in a physically meaningful way instead of epoch-wise estimation. Especially kinematic Single Point Positioning (SPP) will benefit from such an approach called receiver clock modeling (RCM) (Weinbach 2012). Recent developments of low-priced, low power consuming (<120 mW) miniaturized atomic clocks (MACs), primarily Chip Scale Atomic Clocks (CSACs), allow for usage in kinematic GNSS applications. Thus, replacing the receiver's internal oscillator by one of these much more stable external frequency standards opens up the possibility of RCM, and thereby improved positioning.

## Characterization of Miniaturized Atomic Clocks

### Clock comparisons at PTB

In order to correctly model the behavior of an external frequency standard in GNSS data analysis its stability has to be known. Typically, the frequency stability of such a device can be determined by comparison with a frequency standard of at least one magnitude higher stability.

For the present study we use three different miniaturized atomic clocks:

- ▶ Jackson Labs LN CSAC
- ▶ Symmetricom Quantum™ SA.45s CSAC
- ▶ Stanford Research System PRS10

At Physikalisch-Technische Bundesanstalt (PTB), these devices' 10 MHz output signals were compared to the phase of an active Hydrogen Maser (VREMYA-CH VCH-1003A) by means of a phase comparator with a selectable sampling interval of 1 s or 100 s. The 1 s measurements were carried out for a couple of hours and the 100 s measurements for more than a week, respectively. The following results were obtained:

- |  |  |   |
|--|--|---|
| <b>Jackson Labs LN CSAC</b>                            | <b>Symmetricom CSAC</b>                            | <b>SRS PRS10</b>                              |
| ▶ Minimal freq. offsets (1E-11) and drifts (1E-12/day) | ▶ Frequency offset comparable to Jackson Labs CSAC | ▶ Frequency offset around 1E-12               |
| ▶ Frequency deviations: 2E-10 (CSAC), 2E-11 (OCXO)     | ▶ Strong long-term drift around 5E-12/day          | ▶ Long-term frequency drift approx. 1E-12/day |
| ▶ Issues with OCXO steering                            | ▶ Half as noisy as JL CSAC                         | ▶ Frequency deviations: 2E-12                 |

### Allan Deviations

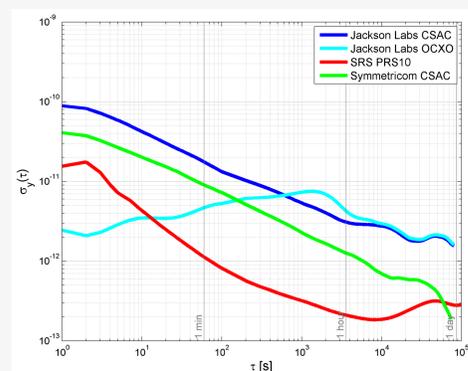


Fig. 2: Overlapping Allan Deviations

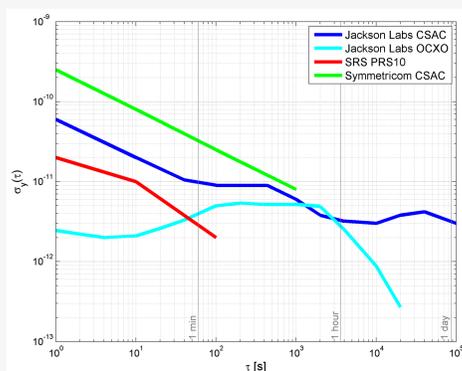


Fig. 3: Manufacturer's Allan Deviations

### Jackson Labs LN CSAC

- ▶ CSAC: White Noise FM up to ca. 1 hour, then Flicker Noise FM
- ▶ CSAC: WFM 1.5 times worse than manufacturer's data
- ▶ OCXO: Typical high short-term stability, performance comparable to manufacturer's data
- ▶ OCXO steering and adjusting to CSAC after approx. half an hour

### Symmetricom CSAC

- ▶ Similar noise types like JL CSAC, but ADEV about 5E-11 smaller
- ▶ Performance more than 5 times better than manufacturer's data
- ▶ Flicker Floor (FFM) after roughly three hours

### SRS PRS10

- ▶ Fluent passages of noise types
- ▶ White Noise PM to FM after approx. 15–20 s
- ▶ Short period of FFM around  $\tau = 2$  hours, followed by Random Walk FM

### Derivation of $h_\alpha$ -coefficients

Before applying in GNSS data analysis, the computed Allan Deviations must be converted to the so called  $h_\alpha$ -coefficients, and thereby, generalized to some extent (Barnes et al. 1971).

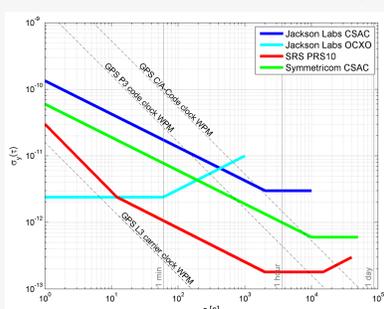


Fig. 4: Course of the derived  $h_\alpha$ -coefficients

- ▶ White Noise PM:  $h_2 = \sigma_y^2(\tau) \cdot \frac{4\pi^2\tau^2}{3f_0}$
- ▶ White Noise FM:  $h_0 = \sigma_y^2(\tau) \cdot 2\tau$
- ▶ Flicker Noise FM:  $h_{-1} = \frac{\sigma_y^2(\tau)}{2 \ln 2}$
- ▶ Random Walk FM:  $h_{-2} = \sigma_y^2(\tau) \cdot \frac{3}{2\pi^2\tau}$

Oscillator type	$h_2$	$h_0$	$h_{-1}$	$h_{-2}$
Jackson Labs CSAC	–	$3.6 \cdot 10^{-20}$	$6.5 \cdot 10^{-24}$	–
Jackson Labs OCXO	–	–	$4.2 \cdot 10^{-24}$	$1.5 \cdot 10^{-26}$
SRS PRS10	$3.5 \cdot 10^{-28}$	$1.4 \cdot 10^{-22}$	$2.3 \cdot 10^{-26}$	$3.3 \cdot 10^{-31}$
Symmetricom CSAC	–	$7.2 \cdot 10^{-21}$	$2.6 \cdot 10^{-25}$	–

Tab. 1: Numerical values of the derived  $h_\alpha$ -coefficients in our experiment

## Pseudo-kinematic Single Point Positioning

### Measurement configuration

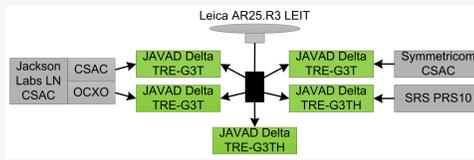


Fig. 5: Measurement configuration of static experiment

- ▶ Leica choke ring antenna AR25.R3 with radome
- ▶ 5 high-end receivers of same type
  - ▶ 4 connected to external atomic clocks
  - ▶ 1 driven by internal quartz oscillator

### Concept of Receiver Clock Modeling

Fig. 4 shows GPS code and carrier clock noise, derived from the observation noise of code and phase observations (1% of the signal's wavelength) and the influence of their elevation-dependent weighting, and modeled as WPM over time (Weinbach 2012). Hence, the intersection points between the dashed lines and the ADEV curves define the maximal time intervals for physically meaningful RCM. Within these intervals the receiver clock error is larger than the stability of the external clocks so that their random frequency fluctuations cannot be resolved by GNSS observations. Obviously, all investigated clocks are well suited for code-based RCM over time intervals of at least one minute.

### Data Analysis

- ▶ Reference coordinates: 3 days PPP solution from *Bernese GNSS Software 5.2* (Dach 2012)
- ▶ Code-based SPP with *IfE GNSS MATLAB Toolbox*
  - ▶ Forward Extended Kalman Filter (EKF)
  - ▶ Ionosphere-free linear combination P3
  - ▶ Final orbits and clock solutions from ESA
  - ▶ Satellite DCBs from CODE
  - ▶ Troposphere: VMF1
  - ▶ RCM over time intervals of 30 s using model of Van Dierendonck et al. 1984

### Results

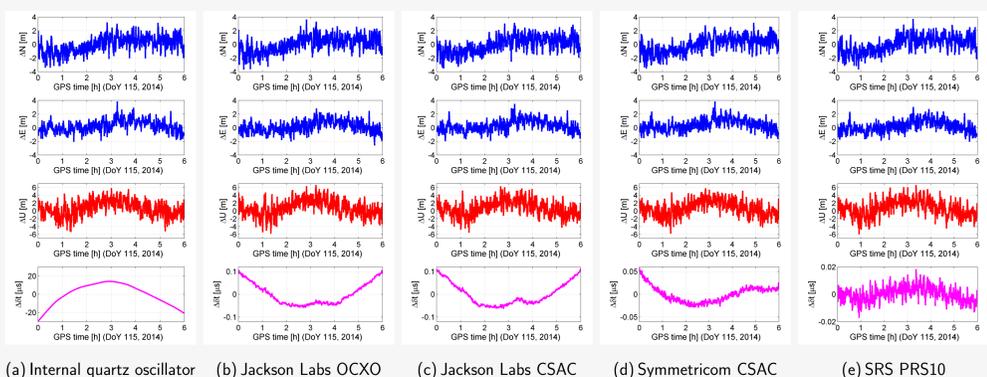


Fig. 6: Topocentric coordinates  $\Delta N$  (North),  $\Delta E$  (East),  $\Delta U$  (Up) w.r.t. reference coordinates and **non-modeled** receiver clock error  $\Delta\delta t$  after straight line fit: (a) Rx driven by internal oscillator, (b)–(e) connected to external frequency standard. Note the different scalings for  $\Delta\delta$ .

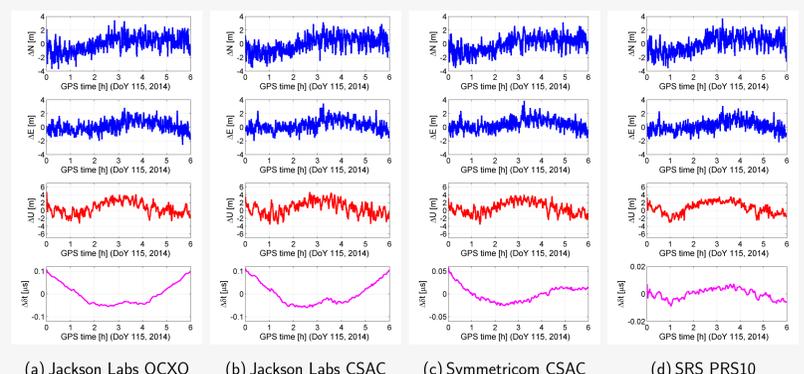


Fig. 7: Topocentric coordinates  $\Delta N$  (North),  $\Delta E$  (East),  $\Delta U$  (Up) w.r.t. reference coordinates and **modeled** (Van Dierendonck et al. 1984) receiver clock error  $\Delta\delta t$  after straight line fit (cf. Fig. 6). Note the different scalings for  $\Delta\delta$ .

- ▶ Application of external clock w/o modeling leads to smaller clock estimates but has no effect on the height coordinates (Fig. 6)
- ▶ RCM smoothes clock & height estimates and reduces the height RMS error (RMSE) (Fig. 7): JL OCXO 20.9%, JL CSAC 15.2%, Symmetricom CSAC 19.6%, SRS PRS10 26.2%
- ▶ Using individual values for RCM has significant influence on SPP height precision
  - ▶ Jackson Labs CSAC values 1.5 times worse than manufacturer's data  $\rightarrow$  height RMSE degraded 1.7%
  - ▶ Symmetricom CSAC values 5 times better than manufacturer's data  $\rightarrow$  height RMSE improved 8.2%

## Conclusions

- ▶ Individual characterizations of MACs agree with manufacturer's data in principle, although Jackson Labs CSAC performs 1.5 times worse and Symmetricom CSAC performs 5 times better
- ▶ Using MACs and applying RCM in (pseudo-)kinematic GNSS SPP improves height RMSE up to 26.2%  $\rightarrow$  increased precision
- ▶ Distortion of height precision if manufacturer's data and individually values differ too much
- ▶ More stringent RCM required since clock model for EKF only smoothes clock and height estimates

## Acknowledgement

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## References

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- A. van Dierendonck, J. McGraw, R. Brown (1984): Relationship Between Allan Variances and Kalman Filter Parameters. IPTTI proceedings, Greenbelt, MD, pp. 273-293.

