An optical clock network for height system unification: a simulation study

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Chronometric levelling

Einstein's general theory of relativity predicts that clocks tick at different rates if they move with different speeds or are under the influence of a gravitational field. Considering the case of two clocks on Earth (both at rest), the change of clocks' frequencies Δf is proportional to the difference in gravity potential ΔW at both sites (Bjehammar 1985):

$$\frac{\Delta f}{f} = \frac{\Delta W}{c^2} + O(c^{-4}).$$

The height difference ΔH can be obtained by

$$\Delta H \approx \frac{\Delta C}{g} = -\frac{\Delta W}{g},$$

where g is gravity value, C is geopotential number.

Error propagation:

$$1.0 \times 10^{-18} \left(\frac{\Delta f}{f}\right) \sim 0.1\ \mathrm{m^2/s^2}\ (\Delta W) \sim 1.0\ \mathrm{cm}\ (\Delta H)$$

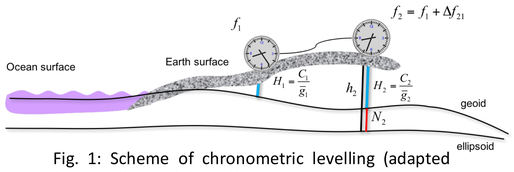


Fig. 1: Scheme of chronometric levelling (adapted from Müller et al. (2018))

This method to obtain heights through the comparison of clocks' frequencies is called **chronometric levelling** (Vermeer 1983). It has advantages to connect distant areas, without being affected by:

- accumulated levelling errors, or
- smoothing effects when combined gravity field models are used.

Clock networks

The comparison of clocks' frequencies between distant sites is approaching the level of $\mathbf{1.0\times10^{-18}}$, benefiting from:

- rapid development of atomic clocks, especially optical clocks;
- dedicated frequency links, e.g., optical fibers.

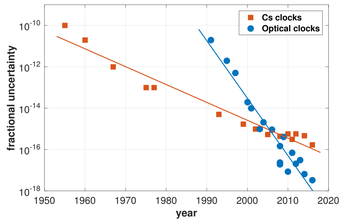


Fig. 2: Evolution of relative frequency accuracy of atomic clocks based on microwaves (Cs clocks) and optical transitions.

Clock networks are becoming a powerful tool for delivering heights over long distances.

Height system unification

For the unification of local height systems, one has to estimate:

- offsets between height datums (up to dm);
- tilts along national levelling lines (1.0 ~ 3.0 cm/100km).

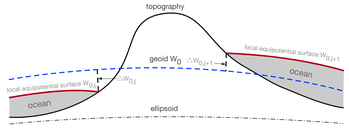


Fig. 3: Offsets between different height datums. The equipotential surfaces for the local height system *j* and *j+1* exhibit discrepancies with respect to the geoid due to the ocean surface topography.

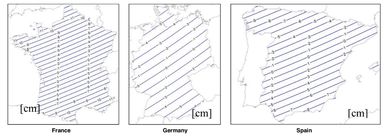


Fig. 4: Tilts in national height systems. The estimated tilts along the longitudinal and latitudinal directions are (1.0, -2.7), (0.9, -1.4) and (0.8, -3.0) cm/100km for France, Germany and Spain. The numbers and figures are taken from Gruber et al. (2014).

Clock networks are well suited to connect different height systems.

Clock networks for height system unification

Simulator

An end-to-end simulation was designed, taking the EUVN/2000 as a priori input. EUVN/2000 was divided into 4 local height systems, i.e., G1, G2, G3, G4, by introducing individual errors. Clock networks (clocks are assumed to be identical and interconnected) are then used for the unification.

Table 1: Introduced errors for each local height system

	G1	G2	G3	G4
random height error (cm)	1.0			
offset c^L (cm)	-18.0	25.0	0	8.0
tilt along lat. a^L (cm/100km)	3.0	-2.0	1.5	-3.0
tilt along lon. b^L (cm/100km)	2.0	3.0	-1.5	-2.0

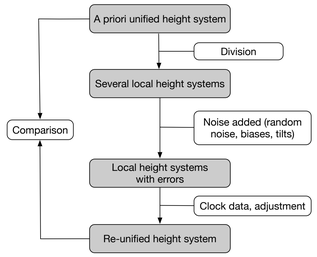


Fig. 5: Scheme of the end-to-end simulation.

Re-unification

$$H_i^L = \frac{C_i^U}{\bar{\gamma}_i} + a^L \Delta X_i^L + b^L \Delta Y_i^L + c^L$$

$$\Delta W_{ij} = W_i^U - W_j^U = -\left(C_i^U - C_j^U\right)$$

Performance of the height unification

For the unification, 4 clocks for each region are used. The accuracy of clock data is assumed as 1.0×10^{-18}.

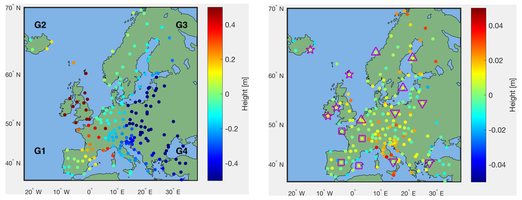


Fig. 6: Introduced height errors (left) and adjusted height errors (right). Symbols (☆, □, △, ▽) represent clocks' positions.

Table 2: RMS of height errors (cm). Introduced errors (before re-unification) and adjusted errors (after re-unification).

	G1	G2	G3	G4
introduced errors	22.81	62.29	15.36	53.65
adjusted errors	0.84	1.19	1.29	1.58

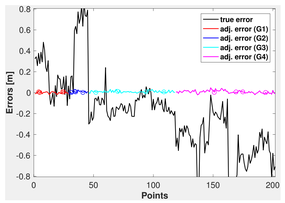
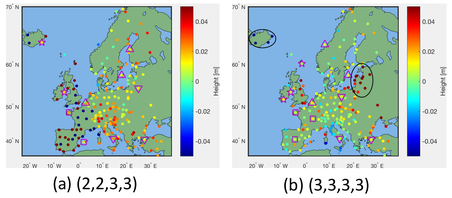


Fig. 7: Height errors for all levelling points of each area.

Test: number of clocks

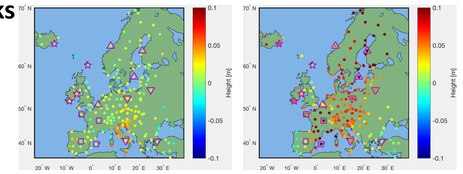


(a) (2,2,2,3) (b) (3,3,3,3)

Fig. 8: Height errors for the re-unified system. Different choices on the number of clocks were tested.

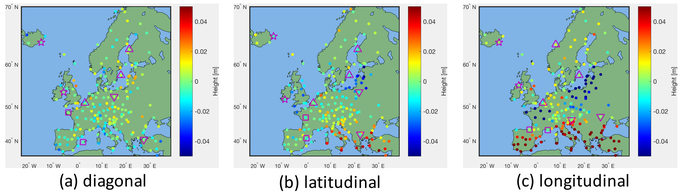
Test: accuracy of clocks

Fig. 9: Height errors for the re-unified system. Clock data with different accuracies were tested.



(a) 5.0×10^{-18} (b) 20.0×10^{-18}

Test: distribution of clocks



(a) diagonal (b) latitudinal (c) longitudinal

Fig. 10: Height errors for the re-unified system. Different spatial distributions of the clocks were compared based on the case where height offsets and tilts along only the latitudinal direction were considered.

Conclusions

- Clock networks show great potential for height system unification.
- Three or four clocks for each region are sufficient for the unification.
- Clocks should be properly arranged to sense the tilts where necessary.
- Clocks with poorer performance can unify the height systems at a certain level.

Future perspectives

Clocks might be used for the realization of an international height reference system (IHRS). More details about IHRS refer to Ihde et al. (2017).

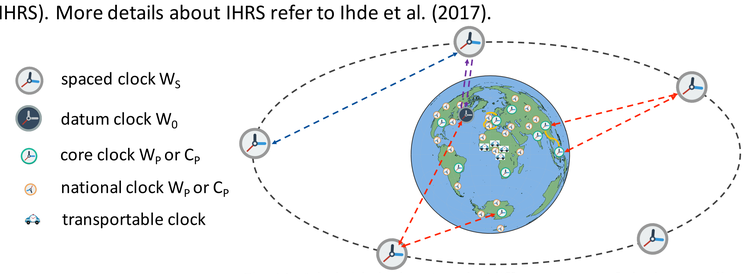


Fig. 11: A hybrid clock network (different types of clocks as well as various frequency link techniques) for the realization of an international height reference system.

Acknowledgements

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