# Benefits from GNSS receiver clock modeling for Low Earth Orbiters

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## Motivation

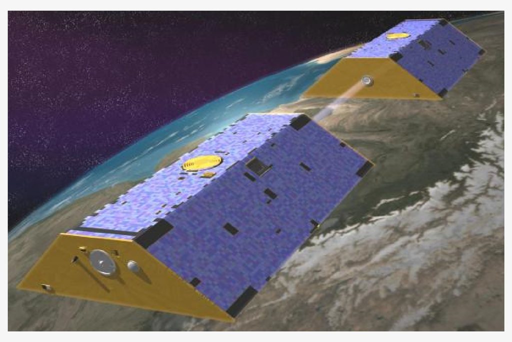

Figure 1: artist's concept of the two GRACE satellites in orbit

- 15 years of Earth's gravity field determination from space
- Low Earth Orbiter (LEO): satellite in ca. 500 km altitude
- several sensors on-board of the LEO satellites
- precise LEO positions are mandatory for gravity estimation
- also absolute timing of all sensor data is necessary
- both can be achieved by using GNSS signals
- errors in Precise Orbit Determination (POD) directly transfer into the gravity field solution
- **two methods of POD for gravity field recovery:** reduced-dynamic orbits, kinematic orbits

## Approaches for precise orbit determination

- **reduced-dynamic orbits:** positions from GNSS data combined with physical force models
  - ⊕ very precise orbits with standard deviations for the coordinates of ca. 3 cm
  - ⊖ depends on introduced force models
- **kinematic orbits:** using GNSS data and attitude information from star camera data
  - ⊕ free of force models, therefore good ability for gravity field determination
  - ⊖ challenges in GNSS positioning

## Concept and challenges of kinematic orbits with PPP

- estimation with Extended Kalman Filter (EKF) or Least-Squares Adjustment (LSA)
- no tropospheric signal delay at LEO altitude
- orientation of LEO's GNSS antenna from star cameras
- reduced-dynamic orbits as reference solution for position comparison
- **remaining challenges:** dynamic ionospheric conditions, nearfield multipath, PCVs, short observation times and small number of GNSS satellites, large number of phase ambiguities

## Concept of Receiver Clock Modeling (RCM)

- **conventional case:** estimation of receiver clock error for every observation epoch
- **RCM:** modeling the receiver clock behavior instead of estimating the clock error
- clock modeling is feasible as long as the Allan deviation of the atomic clock is smaller than the white noise of GNSS phase observations (figure 2)
- RCM for kinematic GRACE positions is possible due to the Ultra Stable quartz Oscillator (USO) on-board

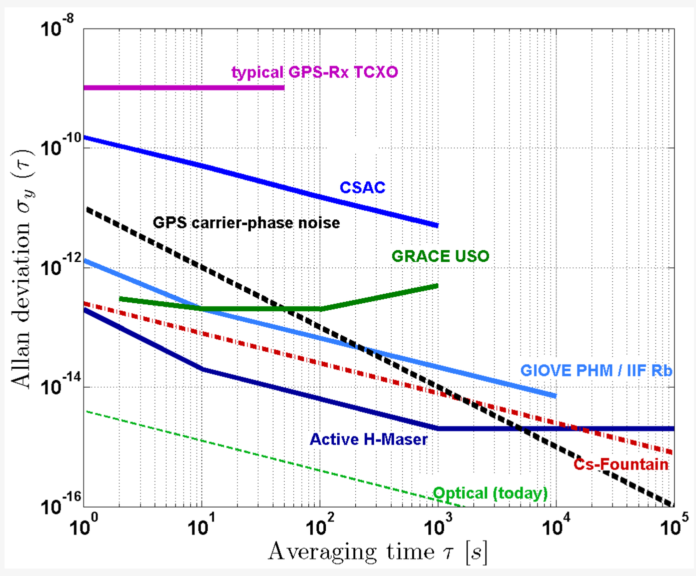


Figure 2: Allan deviation of atomic clocks (Weinbach, 2012)

**Extended Kalman Filter:**

- process noise matrix $Q_w$ with $h_\alpha$ coefficients of the atomic clock (van Dierendonck et al., 1984)
- $$Q_w = \begin{bmatrix} \frac{h_0}{2}\Delta t + 2h_{-1}\Delta t^2 + \frac{2}{3}\pi^2 h_{-2}\Delta t^3 & \frac{h_0}{2} + 2h_{-1}\Delta t + \frac{2}{3}\pi^2 h_{-2}\Delta t^2 \\ \frac{h_0}{2} + 2h_{-1}\Delta t + \frac{2}{3}\pi^2 h_{-2}\Delta t^2 & \frac{h_0}{2\Delta t} + 4h_{-1} + \frac{8}{3}\pi^2 h_{-2}\Delta t \end{bmatrix}$$

**Least-Squares Adjustment:**

- modeling the clock behavior through a piece-wise linear polynomial with coefficients time offsets $o_i$ and frequency offsets $\delta f_i$ (figure 3)
- $\delta t_i = o_i + \delta f_i \cdot (t - t_i)$
- the frequency stability of the atomic clock restricts the length $\Delta t$ of one polynomial part, called the clock modeling interval

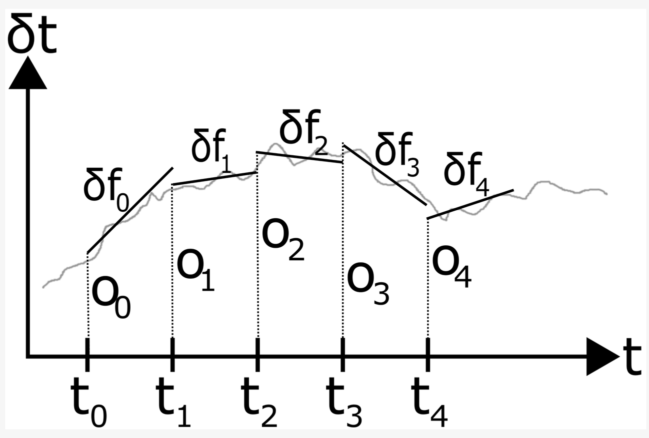


Figure 3: RCM with piece-wise linear polynomials

## Applying RCM for GRACE's GPS data

- GRACE: Gravity Recovery And Climate Experiment, two LEOs at ca. 480 km height (figure 1)
- GPS L1 and L2 observations and reduced-dynamic orbit positions available from JPL
- our simulated observations are based on geometrical distances between GPS and GRACE satellites with P3, L3 observation noise added

**Gain for observation geometry and formal standard deviations:**

- strengthened observation geometry due to smaller DOP values (figures 4 to 6)

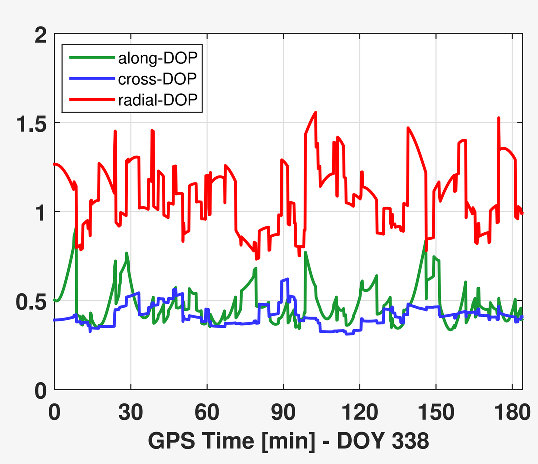


Figure 4: without RCM

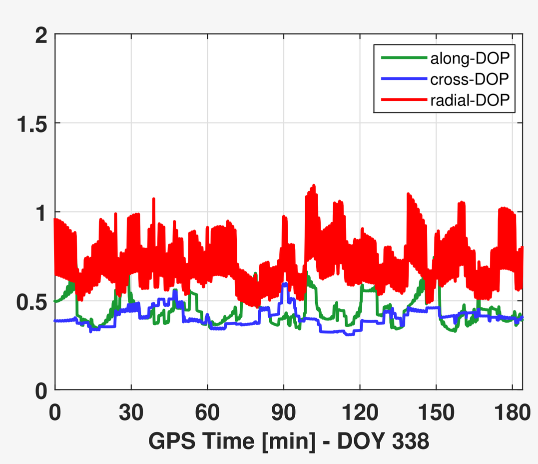


Figure 5: $\Delta t = 60$ s

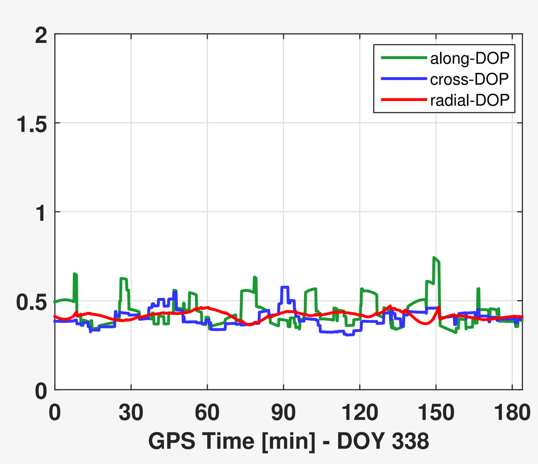


Figure 6: $\Delta t = 2760$ s

- the longer the interval, the smaller the positive correlation of the coordinates among each other
- coordinates of consecutive epochs are linked with a common clock parameter

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## Applying RCM for GRACE's GPS data

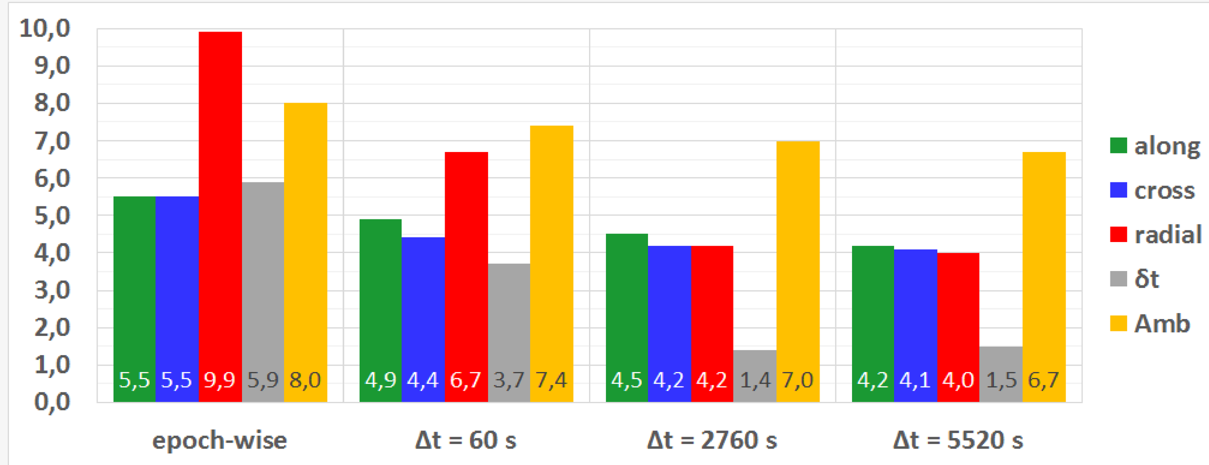


Figure 7: formal mean standard deviation of estimated parameters in mm for different lengths of $\Delta t$ using P3 and L3 (GRACE B, 3rd Dec. 2012 (DOY 338))

**Impact on ambiguity correlations:**

- $\Delta t = 60$ s: ambiguity correlation from +20.5% to +99.6%, mean +62.7% (figure 8)
- $\Delta t = 2760$ s: ambiguity correlation from +32.2% to +99.6%, mean +70.2% (figure 9)
- $\Delta t = 5520$ s: ambiguity correlation from +35.5% to +99.6%, mean +75.1% (figure 10)

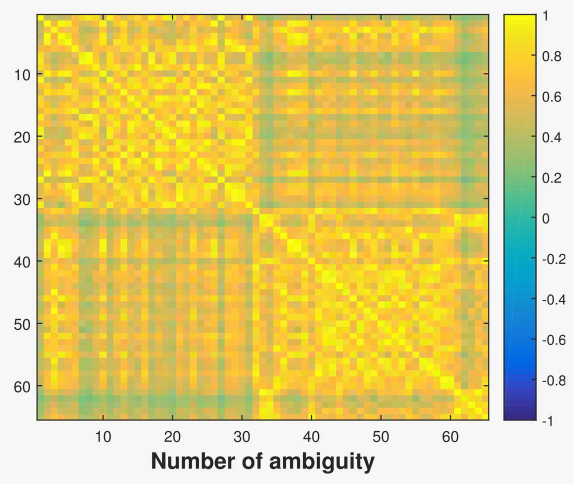


Figure 8: $\Delta t = 60$ s

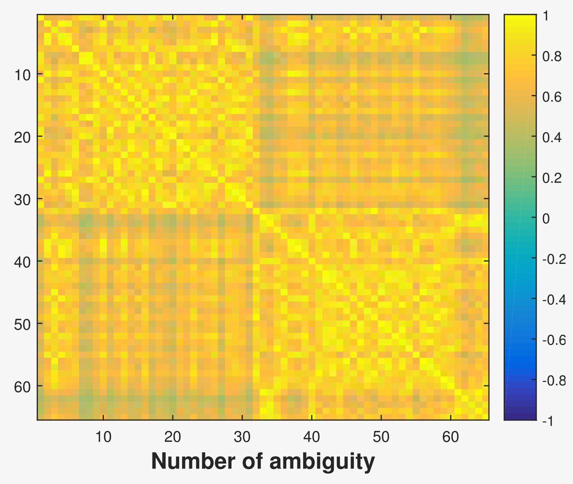


Figure 9: $\Delta t = 2760$ s

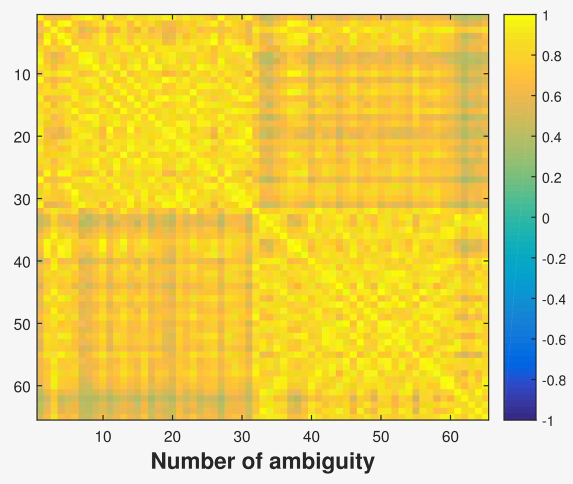


Figure 10: $\Delta t = 5520$ s

- columns for time offsets and ambiguities in design matrix A are linear dependent
- code observations rectify the column singularity, but even the P-Code has a much higher observation noise compared to the phase observations
- **idea:** consider the receiver clock time offsets as parts of the unknown phase ambiguities

## Kinematic PPP with phase only and parameter lumping

- assume one constant ambiguity $N$ per GNSS satellite per continuous observation arc
- $\Phi = \rho + ... + c \cdot \delta t_i + \lambda \cdot N$
- RCM with time offset $o_i$ and frequency offset $\delta f_i$
- $\Phi = \rho + ... + c \cdot (o_i + \delta f_i \cdot t) + \lambda \cdot N$
- $\Phi = \rho + ... + c \cdot \delta f_i \cdot t + c \cdot o_i + \lambda \cdot N$
- lumping all time offsets $o_i$ and the ambiguity $N$ together in one parameter $N^*$
- $\Phi = \rho + ... + c \cdot \delta f_i \cdot t + N^*$ with $N^* = \lambda \cdot N + c \cdot o_0$

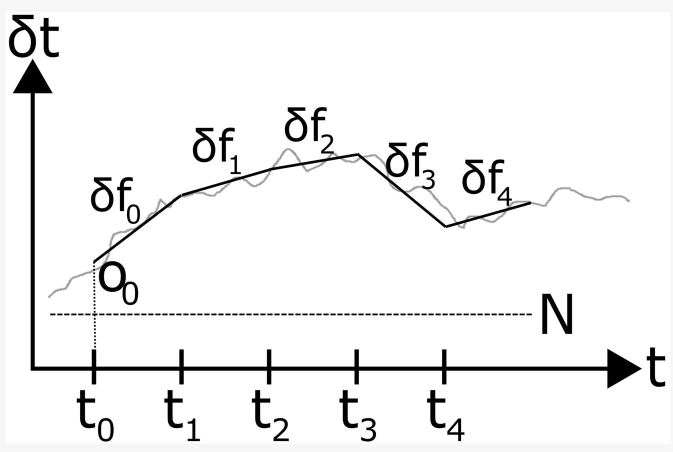


Figure 11: RCM with parameter lumping

- no column singularity between clock offsets and ambiguities, no need of code observations
- kinematic PPP only with high accurate phase observations possible

**Gain for observation geometry and formal standard deviations:**

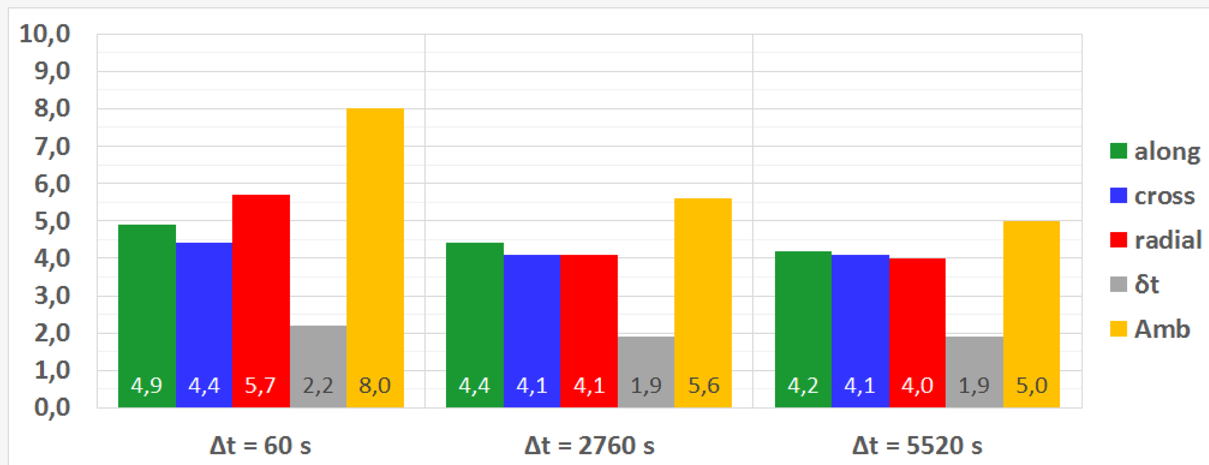


Figure 12: formal mean standard deviation of estimated parameters in mm for different lengths of $\Delta t$ using L3 only (GRACE B, 3rd Dec. 2012 (DOY 338))

**Impact on ambiguity correlations:**

- $\Delta t = 60$ s: ambiguity correlation from +9.8% to +99.8%, mean +63.3% (figure 13)
- $\Delta t = 2760$ s: ambiguity correlation from -78.1% to +99.6%, mean +50.9% (figure 14)
- $\Delta t = 5520$ s: ambiguity correlation from -70.1% to +99.5%, mean +53.5% (figure 15)

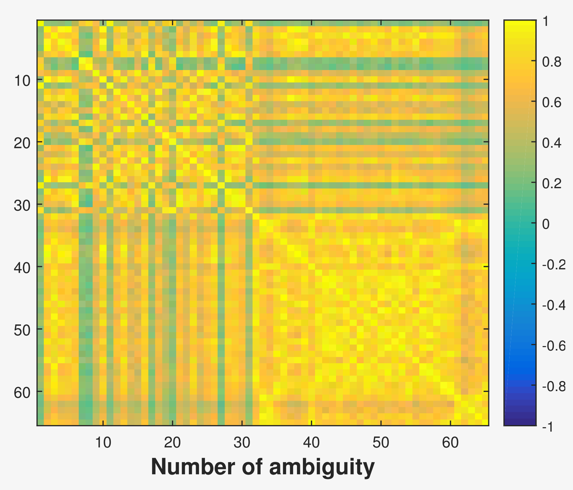


Figure 13: $\Delta t = 60$ s, phase only

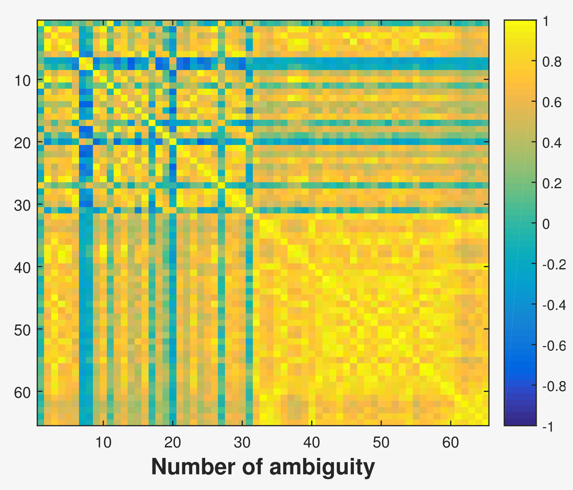


Figure 14: $\Delta t = 2760$ s, phase only

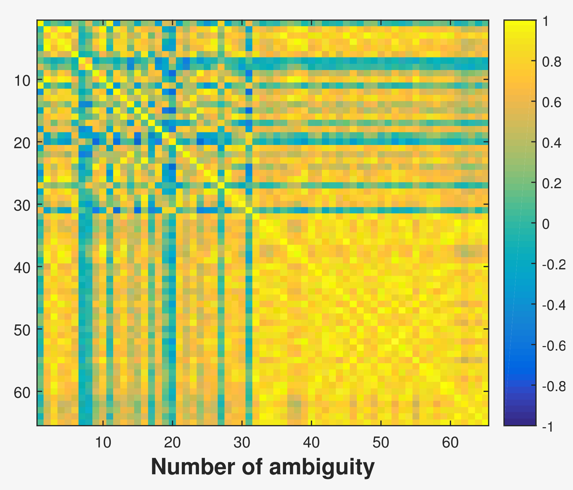


Figure 15: $\Delta t = 5520$ s, phase only

## Conclusion

- RCM for LEOs leads to significantly smaller formal standard deviations for the radial coordinates, receiver clock errors and ambiguities
- new concept of kinematic PPP with phase only and parameter lumping can reach similar improvements in the coordinate domain, even more accurate results for the receiver clock error and the ambiguities with different correlations of the ambiguities among each other

**References**

**Weinbach (2012)** Improved GRACE kinematic orbit determination using GPS receiver clock modeling. In: *GPS Solutions, 17(4):511-520.*
**van Dierendonck et al. (1984)** Relationship between Allan variances and Kalman filter parameters. In: *Proceedings of the 16th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, Greenbelt, MD, pp. 273-293.*

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