Evaluation of Terrestrial Gravity Data by Independent Global Gravity Field Models

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Summary. The new CHAMP and GRACE global gravity field models provide a significantly improved long wavelength gravity spectrum. These satellite-only models are therefore a good basis for studying long wavelength errors of the terrestrial gravity data, as they can be considered as a completely independent source of information.

In this contribution, the models from the CHAMP and GRACE mission as well as EGM96 are used for the evaluation of a terrestrial gravity data set for Europe. The differences are examined both geographically and spectrally. Different techniques are applied for the evaluation, including spherical harmonic expansions, degree variances and the multiresolution analysis based on spherical wavelets. All techniques confirm the existence of small long wavelength errors in the terrestrial gravity data. The reason for such errors may be various, e.g., lacking or poor quality gravity data in some regions, or effects of datum inconsistencies.

Key words: gravity anomaly errors, degree variances, spherical harmonics, multiresolution analysis

1 Introduction

Within the framework of the European Geoid Project, the Institut für Erdmessung (IfE) has collected about 3 million terrestrial gravity data and about 700 million terrain data for Europe and the surrounding marine areas for the computation of the geoid model EGG97, cf. [3]. The original detailed $1' \times 1.5'$ gravity grids include residual terrain reductions (RTM). For this study, these data were averaged in $90' \times 90'$ blocks to filter out high frequency effects. As the individual gravity data sources are coming from different national agencies, it is likely that different standards were used for the data processing. Therefore, small systematic errors may exist in some of the sources. Possible systematic error sources affecting terrestrial gravity data were studied in detail in [5], with the largest components coming from inconsistencies in the gravity and (horizontal and vertical) position reference systems. Moreover, a study on the effect of such datum inconsistencies on European geoid computations is given in [2].

The new satellite gravity field missions CHAMP and GRACE have improved the knowledge of the long wavelength spectrum of the gravity field significantly during the last years. They can therefore be used to identify systematic errors in the terrestrial gravity data at long wavelengths. In the following, the models EIGEN-1S and EIGEN-2 from the CHAMP mission (cf. [6] and [8]), EIGEN-GRACE01S and GGM01S from the GRACE mission (cf. [7] and [1]), and EGM96 as the standard high-degree model are used for the evaluation of the terrestrial data. From all models, $90' \times 90'$ mean gravity anomaly grids were computed up to degree and order 120 for the area covered by the EGG97 data set. The evaluation techniques include spherical harmonic expansions, degree variances, and the multiresolution analysis (MRA) with spherical wavelets, allowing the localization of features in the frequency and space domain.

2 Evaluation Techniques

Within the evaluation process, gravity anomaly differences between the terrestrial data and the global models as well as between different global models were computed and analysed. The first technique uses *RMS degree variances*, which were computed from spherical harmonic expansions of the difference grids (up to $\ell_{\max} = 120$). As the grids are only defined for a small part of the Earth's surface, a scaling of the RMS degree variances by a factor of $\frac{4\pi}{A}$ was done, where A is the area size on the unit sphere. This leads to degree variances that have comparable magnitudes as those derived from planar approximation power spectral densities (PSDs), for details cf. [9]. For a geographical view of the difference patterns, i.e. in the space domain, a *spherical harmonic synthesis with varying maximum degree* $\ell_{\max}$ was performed.

Another technique, which allows a combined spectral and spatial evaluation, is the *multiresolution analysis* (MRA). Spherical wavelet functions Ψ were used. They only depend on the spherical distance and are defined by a Legendre series, see [4]. For the MRA, the mother wavelet is contracted and dilated in fixed steps, resulting in discrete scales j with corresponding wavelet functions Ψ_j. The Legendre coefficients for each function determine the spectral behavior. The range of degrees, where the coefficients are unequal zero, is the range of the spectral localization for the scale j. Table 1 shows the spectral bands localized by the smoothed Shannon wavelet (SSW), which is used in this study.

3 Evaluation Results

Fig. 1 shows the RMS degree variances for 4 selected difference data sets as well as the errors for 2 of the involved global models. All other cases are not shown here due to space restrictions. The differences between the terrestrial data and the CHAMP and GRACE models show a significant increase around degree 45 and 105, respectively, where the satellite-only models do

Table 1. Frequency localization of the smoothed Shannon wavelet

scale j	spectral band		
0	0	–	1
1		2	
2	3	–	4
3	5	–	8
4	9	–	16
5	17	–	32
6	33	–	64
7	65	–	128

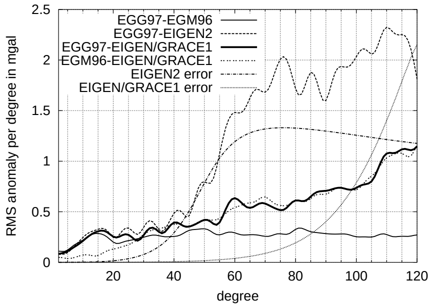

Fig. 1. RMS anomaly degree variances.

not have the full power due to attenuation effects. Considering also the errors of the data sets involved, one can find that the differences are exceeding the corresponding error estimates by a factor of two up to about degree 35 for the CHAMP models and up to about degree 90 for the GRACE models, respectively. In these comparisons, the EIGEN-2 model is performing slightly better than the EIGEN-1S model, while the two GRACE models (EIGEN-GRACE01S and GGM01S) show very similar results. It should also be noted that the agreement of the terrestrial data with the EGM96 model is superior to the other models at higher degrees. However, this is probably due to the fact, that terrestrial gravity data is already included in the EGM96 model. At present, the GRACE models should be the best source of information to determine long wavelength errors of the terrestrial data, allowing spherical harmonic expansions up to about degree 90.

A geographical view of the difference patterns, derived from the spherical harmonic expansions up to a fixed maximum degree, is given for some selected cases in Figs. 2-5. While the agreement of the two global models EIGEN-2 and EGM96 is very good up to degree 20, the differences between the EGG97 data and the EIGEN-2 model show a RMS value of 0.8 mgal and maximum values up to ± 3 mgal in the Mediterranean Sea, Eastern Europe, and Greenland. All these areas are known for their weak data quality. On the other hand, the agreement between the terrestrial data and EIGEN-2 is significantly better over the continental parts of central, western and northern Europe. These regions are covered by high quality gravity data, and only small differences, possibly resulting from datum inconsistencies, are found. All these features are even more pronounced in the comparisons with the highly accurate EIGEN-GRACE01S model. The corresponding differences up to spherical harmonic degree 50 are shown in Figs. 4 and 5. In the differences between the EGG97 data and the EIGEN-GRACE01S model, the problem areas show up more clearly with the largest values being about ± 8 mgal in the eastern part of the Mediterranean Sea and Greenland. Again, the discrepancies over central Europe, where high quality data are available, are

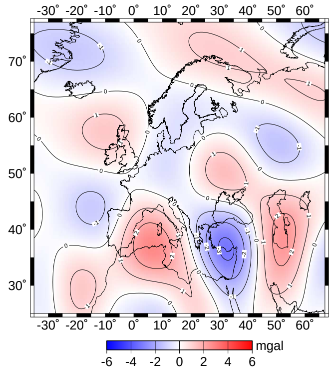

Fig. 2. Differences between EGG97 and EIGEN-2 up to $\ell_{\max} = 20$.

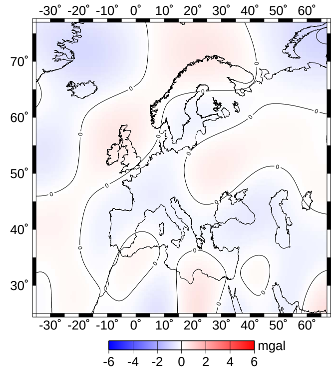

Fig. 3. Differences between EGM96 and EIGEN-2 up to $\ell_{\max} = 20$.

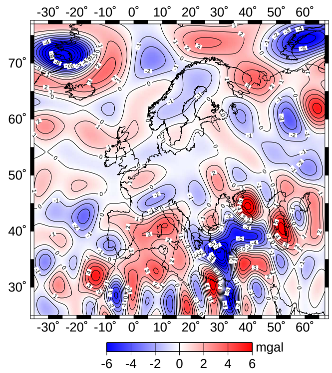

Fig. 4. Differences between EGG97 and EIGEN-GRACE01S up to $\ell_{\max} = 50$.

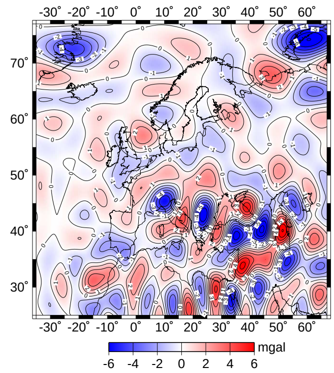

Fig. 5. Differences between EGM96 and EIGEN-GRACE01S up to $\ell_{\max} = 50$.

very small and below 1-2 mgal. Over northern France, a small positive offset exists, while in southern France and over Switzerland a small negative offset can be seen. These features may be due to vertical and horizontal datum problems, e.g., the French and Swiss national height systems are offset with respect to UELN by several dm. Moreover, also the differences between the EGM96 and EIGEN-GRACE01S models up to degree 50 attain larger values, especially over the Alps and Southeast Europe, where the data used in the EGM96 development are probably weak.

Considering the results of the spherical harmonic analysis, the most interesting features of the MRA should reside at the scales 4 to 6 of the SSW (cf. Table 1). The results of the MRA are illustrated exemplarily for scales 4 and 5, again using the differences EGG97/EIGEN-2 and EGM96/EIGEN-2 (see Figs. 6-9). Moreover, an analysis with the GRACE models was done. The results showed a very good agreement with the EIGEN-2 analysis for scales up to 5. Due to space restrictions, no details are provided on the GRACE

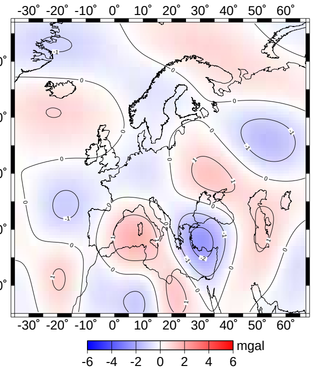

Fig. 6. Differences between EGG97 and EIGEN-2 from MRA, SSW, scale 4.

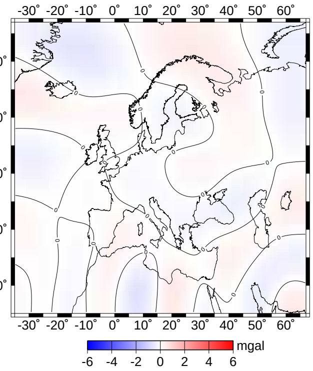

Fig. 7. Differences between EGM96 and EIGEN-2 from MRA, SSW, scale 4.

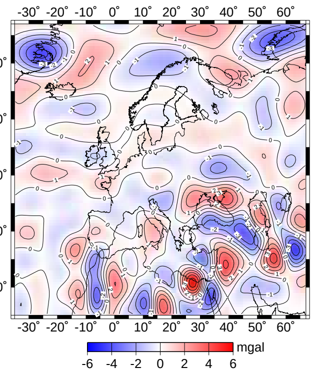

Fig. 8. Differences between EGG97 and EIGEN-2 from MRA, SSW, scale 5.

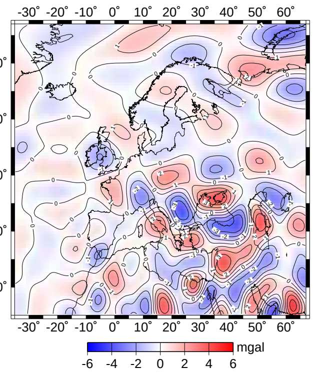

Fig. 9. Differences between EGM96 and EIGEN-2 from MRA, SSW, scale 5.

results. At scale 4 (see Figs. 6 and 7), representing the spectral band from $\ell = 9$ to 16, the differences between EGG97 and EIGEN-2 reach extreme values of ± 3 mgal and a RMS of about 0.6 mgal. The largest differences are located in the Mediterranean Sea and in the Russian plains west of the Ural. Again, the differences between the global models EGM96 and EIGEN-2 are very smooth with a RMS around 0.2 mgal, and they do not show patterns like the EGG97 comparisons. At scale 5 ($\ell = 17$ to 32) the results are more diverse (see Figs. 8 and 9). The differences between EGM96 and EIGEN-2 show a larger RMS around 1.1 mgal, and the maximum values reach about ± 4 mgal. As the analysis with the GRACE model showed similar features, this indicates areas with weak data sets in the EGM96 development (e.g., Southeast Europe). In the EGG97 comparison, small differences are found for most parts of contintental Europe. Only in Southeast Europe, Africa, and at the extreme northern latitudes, larger differences up to about ± 5 mgal are found. Again these are the areas with lacking or poor quality gravity data.

4 Conclusions

In this paper, the long wavelength quality of terrestrial gravity anomalies in Europe and its surrounding seas was investigated using the new global gravity field models from the CHAMP and GRACE mission. From spherical harmonic expansions and degree variances significant differences between the terrestrial and the satellite data were found up to degree 35 for CHAMP and up to degree 90 for GRACE. The problem areas with larger differences are the the Mediterranean Sea, Greenland and Eastern Europe, all these areas are known for their weak data quality. In the continental parts of central, northern and western Europe, covered by high quality gravity data, the agreement is much better. Here, only some small effects from datum inconsistencies may exist. The multiresolution analysis (MRA) confirmed the findings from the spherical harmonic analysis.

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