Evaluation of Terrestrial Gravity Data by New Global Gravity Field Models

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Abstract. New global gravity field models derived from the satellite missions CHAMP, GRACE and GOCE are expected to significantly improve the long wavelength gravity spectrum. These models have the ability to study long wavelength errors of the terrestrial gravity data, which have been found in several global and regional comparisons.

First, the improvement in the modelling of the long wavelength gravity field components will be studied based on the EGM96 model and the EIGEN-1S (CHAMP) model. Second, the terrestrial gravity data for Europe, collected within the frame of the International European Geoid Project, will be utilised to study the long wavelength agreement between the terrestrial data and the global models. The differences are examined both geographically and spectrally. Different techniques are applied for the evaluation, including spherical harmonic expansions, degree variances, 2D Fourier techniques and wavelet transforms. All techniques confirm the existence of small long wavelength errors in the terrestrial gravity data. The reasons for the revealed errors may be various. In some regions the problem can be traced back to lacking or poor quality gravity data, while in other regions we suspect datum inconsistencies or errors in the reference gravity stations.

Keywords. gravity anomaly errors, spherical harmonic analysis, degree variances, multiresolution analysis

1 Introduction

Within the frame of the European Geoid Project, supported by the International Association of Geodesy (IAG), the Institut für Erdmessung (IfE) has collected about 3 million terrestrial gravity data and about 700 million terrain data for Europe and the surrounding marine areas, see Denker and Torge (1998). As the individual data sources are coming from different national agencies, it is likely that different standards were used for the data processing, and therefore small systematic errors may exist in some of the sources. Possible systematic error sources affecting terrestrial gravity data were studied in detail by Heck (1990), with the largest components coming from inconsistencies in the gravity and position (horizontal and vertical) reference systems. Moreover, a study on the effect of such datum inconsistencies on European geoid computations was performed by Denker (2001).

With the launch of the new satellite gravity field missions CHAMP and GRACE, which will improve the quality of global gravity field models especially at long wavelengths, there is the possibility to evaluate systematic errors in the terrestrial data. To date the CHAMP mission has led to a new global gravity field model called EIGEN-1S, which is complete to degree and order 100 and contains selected higher degree terms up to a maximum degree of 119. However, this model is a satellite-only model and therefore it has got full full power only up to about degree and order 35, for details see Reigber et al. (2001, 2002).

The terrestrial gravity anomalies used in this study are identical with the data set used for the computation of the European geoid model EGG97, see Denker and Torge (1998). For the following investigations the detailed grids were merged to $1° \times 1°$ blocks, covering the entire European continent and the adjacent seas (the extent of the study area can be seen from Fig. 3). To account for the different frequency contents of the terrestrial data and the EIGEN-1S model, the high degree effects, not contained in the global model, were removed by the high-degree spherical harmonic model EGM96 ($\ell \geq 120$). This technique is also used within the EGM96 development, see Lemoine et al. (1998). Moreover, it was also tested to filter out the high degree effects by averaging to appropriate block sizes. However, this procedure will not be discussed in the following as it gave nearly identical results.

Because it is difficult to decide which of the data

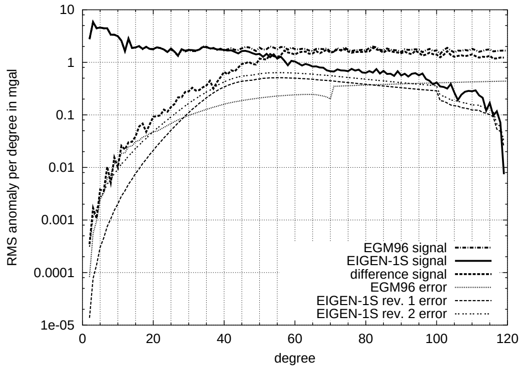

Figure 1: RMS anomaly per degree for EIGEN-1S and EGM96 (signal, error and difference).

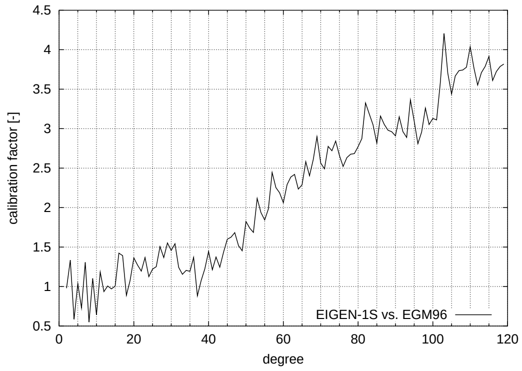

Figure 2: Calibration factor per degree for EIGEN-1S (rev. 2) versus EGM96.

sets is responsible for existing discrepancies, we show also comparisons with the EGM96 model to be used as a reference. Furthermore, we will directly compare the EIGEN-1S and EGM96 models to check the consistency of these global models at long wavelengths. Besides the degree variance approach we will show results from the wavelet technique and the spherical harmonic analysis.

2 Comparison of EIGEN-1S and EGM96

Before evaluating the terrestrial gravity data we compare the two global models EIGEN-1S and EGM96 in order to check the consistency of the coefficients and the corresponding error estimates. At first we study the signal and error anomaly degree variances as well as the difference anomaly degree variances between the two models. All RMS degree variances are shown in Figure 1. The RMS signal of the two models is nearly identical up to degree 40, while the EIGEN-1S satellite-only model is loosing more and more power at higher degrees, see also Reigber et al. (2001, 2002). Regarding the error estimates for the EIGEN-1S model, a revision 1 and 2 exist. The rev. 2 error estimates were derived from the rev. 1 values by down-scaling with a degree-dependent factor. Both versions are shown in Figure 1. The rev. 2 error estimates are expected to be more realistic and are very close to the EGM96 error estimates up to about degree 20, while the EIGEN-1S (rev. 2) values become significantly larger than the EGM96 values above degree 20. The difference anomaly degree variances show a good agreement of both models in the low degrees and are within the magnitude of the error estimates.

Table 1: Gravity anomaly differences in Europe between EIGEN-1S and EGM96 for a varying maximum degree of the spherical harmonic expansion. Units are mgal.

$\ell_{\max}$	Mean	RMS	Min.	Max.
10	0.051	0.112	-0.128	0.348
15	0.065	0.191	-0.396	0.533
20	0.073	0.304	-0.815	0.916
25	0.089	0.524	-1.545	1.762
30	0.086	0.811	-2.389	2.681
35	0.085	1.212	-3.688	4.016
40	0.084	1.657	-4.633	6.314

The consistency of the two models in terms of coefficients and error estimates can also be analysed by the calibration factors introduced e.g. in Lerch et al. (1988). Fig. 2 shows these calibration factors, describing the rates of the model differences and the corresponding error estimates. The calibration factors are close to 1.0 for degrees up to about 40 and increase up to about 3.0 for the higher degrees. This shows that up to degree 40 the model differences and error estimates are consistent, while this is not the case at the higher degrees, the main reason being probably too little power and too optimistic error estimates of the EIGEN-1S model at these degrees.

Another possibility to evaluate the two models is the computation of anomaly difference grids with a varying maximum degree of the spherical harmonic expansion. Here the analysis of the differences is restricted to Europe. The statistics of the differences (Table 1) and the difference plots (Fig. 3) give valuable information about the spectral

Δg (EIGEN-1S - EGM96; l_{max}=30)

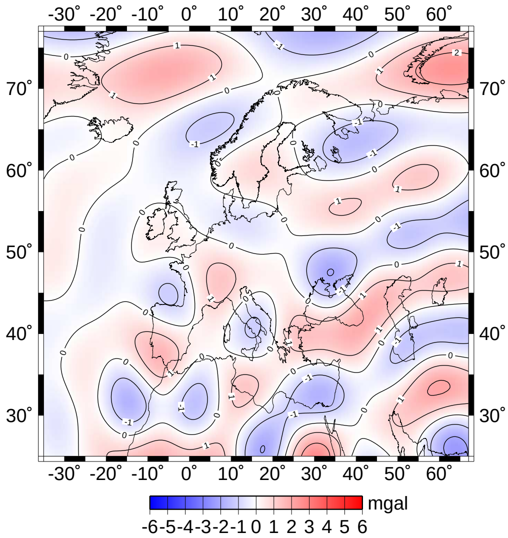

Figure 3: Gravity anomaly differences between EIGEN-1S and EGM96 in Europe up to degree 30. Units are mgal.

and spatial behaviour. The statistics shows that the RMS differences between the two models exceed 1 mgal at about degree 30. Up to degree 10 the RMS difference is only 0.1 mgal, while up to degree 20 we have 0.3 mgal. Figure 3 shows a smooth difference field with maximum variations of ±2.7 mgal at $\ell_{\max} = 30$. Overall, the agreement of the two models is quite good up to degree 20 to 30, and thus both models should be useful for the detection of long wavelength errors in the terrestrial gravity data.

3 Evaluation of the Terrestrial Data

In this section we will evaluate the terrestrial gravity data set (denoted as EGG97 gravity data in the following) by comparison with the two global models EIGEN-1S and EGM96. At first, the anomaly differences are analysed using spherical harmonic expansions, with the coefficients being used to compute the corresponding anomaly degree variances. The RMS anomaly differences per degree were computed by equation (1) and were scaled by the factor $^{4\pi}/_{A}$ (A is the area of the data region on the unit sphere) to consider the regional character of the EGG97 gravity data set. Secondly, the degree variances were computed by planar approximation from the power spectral density PSD of the difference grids using equation (2), where the PSD is computed using the two-dimensional Fourier transform, see Forsberg (1984). The degree variances from the spherical and planar approach are illustrated in Figures 4 and 5.

$$\sigma_\ell = \left[\left(\frac{GM(\ell-1)}{R^2} \right)^2 \sum_{m=0}^{\ell} \left(\delta\bar{C}_{\ell m}^2 + \delta\bar{S}_{\ell m}^2 \right) \right]^{1/2} \quad (1)$$

$$\sigma_\ell = \left[\left(\ell + \frac{1}{2} \right) \frac{1}{2\pi R^2} PSD \left(\frac{\ell + \frac{1}{2}}{2\pi R} \right) \right]^{1/2} \quad (2)$$

From Figures 4 and 5 we can see a reasonable agreement between the spherical and the planar computations. The Fourier technique exposes less details of the spectrum because of the sampling theorem, as it cannot provide a value for every degree ℓ. Both Figures (4 and 5) show a strong increase of the differences involving EIGEN-1S at about degree 35. This goes along with the results of Sect. 2, showing again that the EIGEN-1S model has very little signal above degree 35. In Fig. 4 both differences involving the terrestrial gravity data show some minor peaks at low degrees ($\ell = 16, 24, 32$) which correspond to wavelengths of about 2500, 1700 and 1250 km at the Earth's surface. When comparing the differences in Fig. 4 with the errors of the global models in Fig. 1 and considering also the errors of the terrestrial anomalies, we find that the differences exceed the error estimates by at least a factor of two up to degree 30 and by at least a factor of three up to degree 25. Thus significant long wavelength differences exist between the terrestrial gravity data and both global models up to about degree 25 ... 30. These findings agree well with results obtained by Pavlis (1998, 2000) on a global scale. Possible reasons can be the datum inconsistencies, studied e.g. by Heck (1990) and Denker (2001). However, these effects cannot fully explain the discrepancies found in our study area. Finally, it should be noted that the degree variance approach can only determine the frequencies of the phenomenon, but not the locality.

One can overcome this problem through the spherical harmonic analysis of the differences and subsequent synthesis to a varying maximum degree (as used already in Sect. 2) or by the wavelet analysis of the differences. Both techniques allow a geographical localisation, which is important if we want to find out the areas with systematic errors in the terrestrial data.

The Figures 6–9 show the anomaly differences between the EGG97 gravity data and the global models

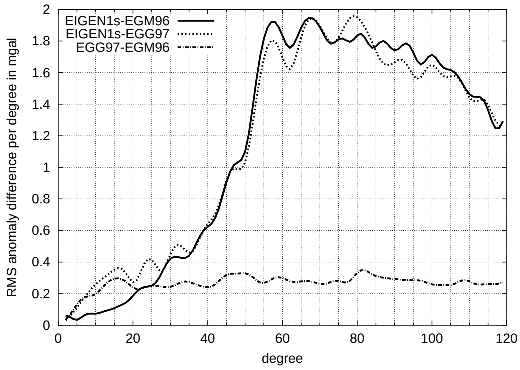

Figure 4: RMS anomaly difference per degree for EIGEN-1S/EGG97/EGM96 according to eq. (1).

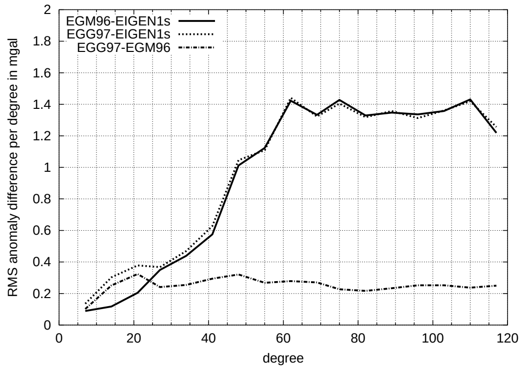

Figure 5: RMS anomaly difference per degree for EIGEN-1S/EGG97/EGM96 according to eq. (2).

EIGEN-1S and EGM96 expanded into spherical harmonics up to degree 20 and 30, respectively. One can see that the differences between the terrestrial gravity data and the EGM96 and EIGEN-1S model are quite similar and significantly larger than the differences between the two models itself. This again suggests significant long wavelength errors in the terrestrial data up to degree 25 ... 30. For $\ell_{\max} = 20$ the maximum differences reach about ± 3 mgal while they reach about ± 5 mgal at $\ell_{\max} = 30$, which is significantly above the differences between the two models itself and the corresponding error estimates. Regions with larger differences include the Eastern and Western Mediterranean Sea, Greenland, Eastern Europe, etc. Some of these regions are known to have weak gravity data.

The second method to reveal spatial information from the difference grids is the wavelet analysis. By applying a planar approximation to the data region we can use a two-dimensional discrete wavelet transform to compute wavelet coefficients at different discrete scales L_i. This is also called multiresolution analysis (MRA). The scales are connected to the frequencies. For each scale the signal is decomposed into an approximation and three details. Extreme values of the detail-coefficients show a large correlation of the signal and the wavelet function at that scale. With the help of the 2D-MRA one can see the locality of a difference, its wavelength and the direction of the structure at the same time. Note that this is only possible for some discrete wavelengths because of Heisenberg's uncertainty relation. For a more detailed introduction to MRA see Jawerth and Sweldens (1994).

For the decomposition shown in Fig. 10, a coiflet wavelet of order 2 was used, see Daubechies (1992). The scales L_1–L_3 represent wavelengths of

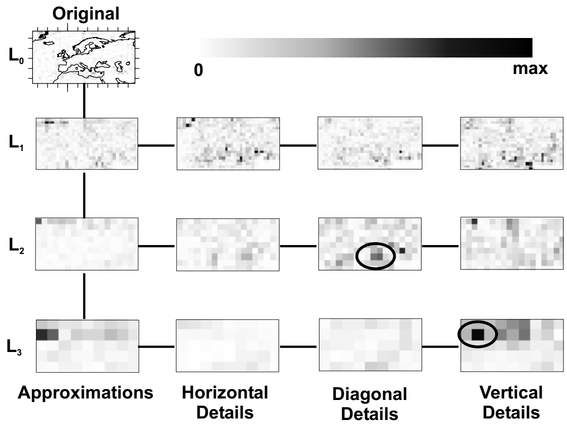

Figure 10: Wavelet decomposition of EGG97/EIGEN-1S differences with coiflet 2 wavelet.

about 460 km, 920 km and 1830 km. One can see many features appearing at the lowest scale L_1, but this corresponds to a degree higher than $\ell = 35$, where EIGEN-1S is not reliable enough to draw conclusions. At scales L_2 and L_3, the Greenland and the Mediterranean Sea regions are detected as problem areas.

4 Summary and Conclusions

In this paper we have shown that the global gravity field models EIGEN-1S and EGM96 are consistent up to about degree 35, while above this degree the EIGEN-1S may have too little signal and too optimistic error estimates. The EIGEN-1S has better accuracy estimates in the degree range 5 to 20.

The evaluation of the terrestrial gravity data for

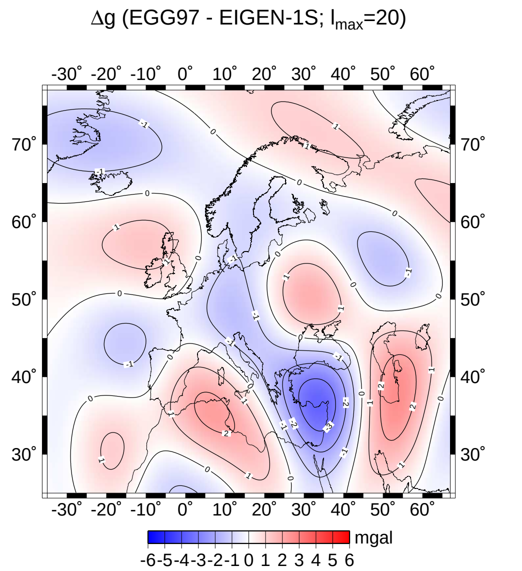

Figure 6: Residuals between EGG97 and EIGEN-1S gravity anomalies up to $\ell_{\max} = 20$. Units are mgal.

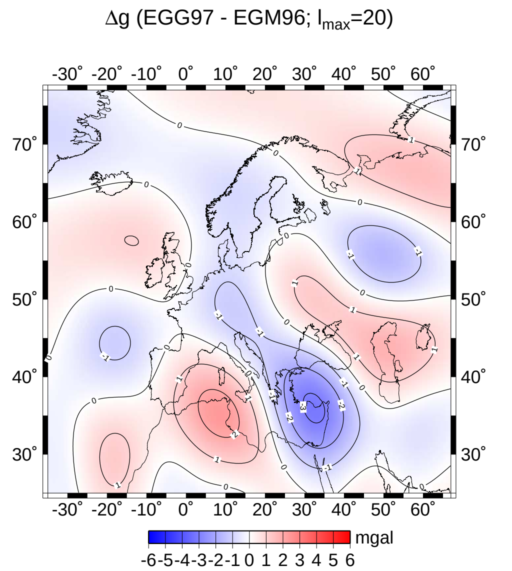

Figure 7: Residuals between EGG97 and EGM96 gravity anomalies up to $\ell_{\max} = 20$. Units are mgal.

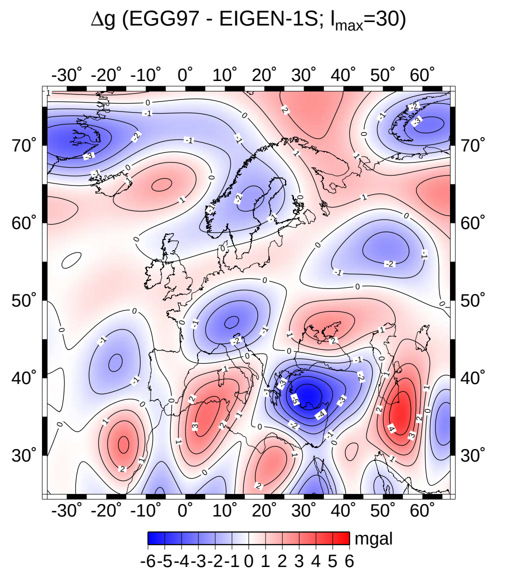

Figure 8: Residuals between EGG97 and EIGEN-1S gravity anomalies up to $\ell_{\max} = 30$. Units are mgal.

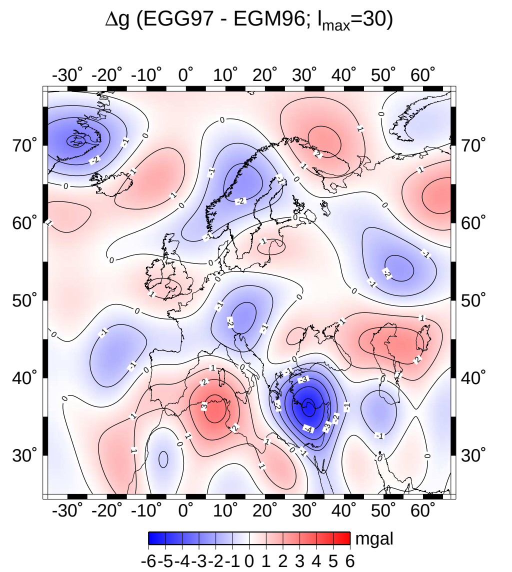

Figure 9: Residuals between EGG97 and EGM96 gravity anomalies up to $\ell_{\max} = 30$. Units are mgal.

Europe by comparison with the EIGEN-1S and the EGM96 global model has confirmed the existence of small long wavelength errors which are significant up to about degree 25 to 30. The spherical harmonic and the wavelet analysis provide informations on regions with larger differences. Such regions are the Eastern and Western Mediterranean Sea, Greenland, Eastern Europe, etc. The reasons for the revealed errors in the terrestrial data are various. In some regions the problem can be traced back to lacking or poor quality gravity data. In other regions we suspect datum inconsistencies or errors in the reference gravity stations. Although possible unconsidered reference system inconsistencies may play an important role, they cannot fully explain the discrepancies found in our study area.

Therefore it is very important to process all gravity data using common standards. For some of the data sets a re-processing may lead to a better long wavelength accuracy. However, if such improvements are not possible because the underlying standards are not known in detail or are not available for different reasons, one should account for long wavelength errors in the terrestrial gravity data in combination solutions either by modelling or weighting techniques or both.

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