Generation of slant tropospheric delay time series based on turbulence theory

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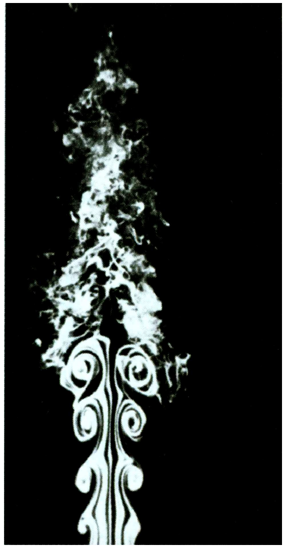
Slant tropospheric delay:

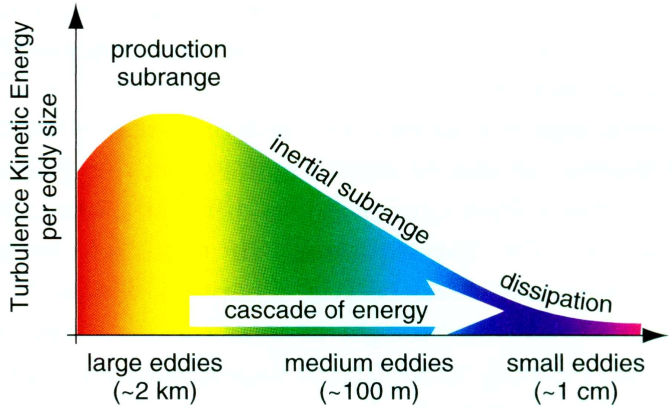
- long-periodic variations:
  - caused by e.g. daily - hourly variations of temperature, pressure, partial pressure of water vapor, ...
  - mean behaviour described by **deterministic** models (e.g., Hopfield, Saastamoinen, ... & mapping functions)
- short-periodic variations (periods of [min] to [sec]):
  - caused by: turbulent flow in atmospheric boundary layer
  - water vapor variations $\Rightarrow$ index of refractivity variations
  - behaviour described **stochastically** ($\Rightarrow$ Turbulence theory)



Wallace, Hobbs (2006): Atmospheric Science

Turbulence theory / 'Wave propagation in turbulent media':

- Spectrum of turbulence kinetic energy:



Wallace, Hobbs (2006): Atmospheric Science

- von Karman spectrum (→ non-stationary process):

$$\Phi_n(\kappa) = \frac{0.033\ C_n^2}{(\kappa^2+\kappa_0^2)^{\frac{11}{6}}} \propto \kappa^{-11/3}, \quad 0 < \kappa < \kappa_S$$

C_n^2	structure constant of refractivity
$\kappa_0 = 2\pi/L_0$	wavenumber corresponding to outer scale length L_0

⇒ Stochastic model of GNSS phase observations (can be regarded as stochastic model of slant delays)

Turbulence theory-based covariance (Schön/Brunner, JGeod 2007 82(1), pp. 47-57):

$$\begin{aligned}\langle \varphi_A^i(t_A),\ \varphi_B^j(t_B)\rangle &= \langle \tau_A^1(t_A), \tau_B^2(t_B)\rangle = \\ &= \frac{12}{5}\frac{0.033}{\Gamma\left(\frac{5}{6}\right)}\frac{\sqrt{\pi^3}\kappa_0^{-\frac{2}{3}}2^{-\frac{1}{3}}}{\sin\varepsilon_A^i \sin\varepsilon_B^j} C_n^2 \\ &\times \int_0^H\int_0^H (\kappa_0 d)^{\frac{1}{3}} K_{-\frac{1}{3}}(\kappa_0 d)\ dz_1\ dz_2\end{aligned}$$

$\varphi_A^i(t_A)$	Phase observation (station A, satellite i, epoch t_A)
C_n^2	Structure constant (characterises strength of turbulence)
L_0	Outer scale length ($\kappa_0 = 2\pi/L_0$)
H	Integration height
ε_A^i	Elevation of satellite i at station A
d	separation between integration points
K	modified Bessel function
Γ	Gamma function

Generation of slant delay variations

Simulation of slant delay variations using EVD of $\boldsymbol{\Sigma}$:

$\mathbf{y} = \mathbf{G}\sqrt{\boldsymbol{\Lambda}}\mathbf{x}$ with

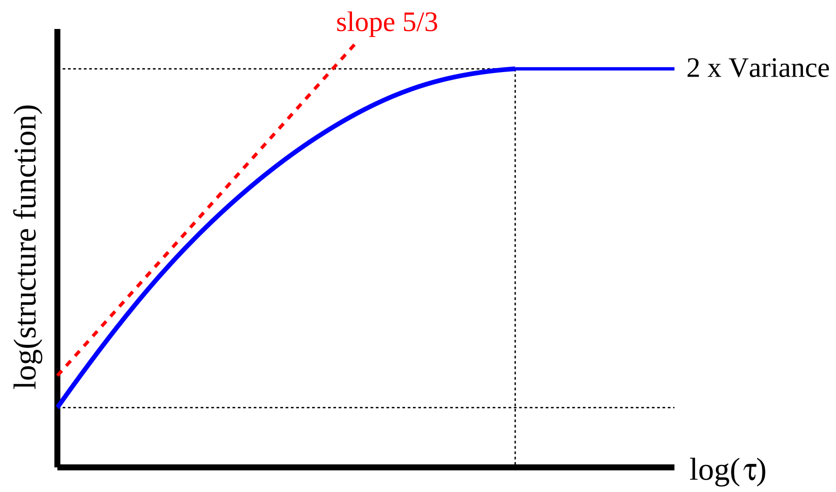
$\mathbf{G}$	eigenvectors of $\boldsymbol{\Sigma}$
$\boldsymbol{\Lambda}$	diagonal matrix, eigenvalues λ_i of $\boldsymbol{\Sigma}$
$\mathbf{x}$	gaussian random vector with $N(0,1)$

$\rightarrow$ **variations** of slant delays (5 realisations)

Stochastic properties:

- Temporal structure function / Variogram:

$$D_n(\tau) = \langle [n(t+\tau) - n(t)]^2 \rangle \propto \tau^{\frac{5}{3}}$$



Analysis objectives

Analysis objectives:

Investigation of impact of parameter variations on:

- Variance-covariance matrices and correlation matrices
- Stochastic behaviour of simulated **variations** of slant delays

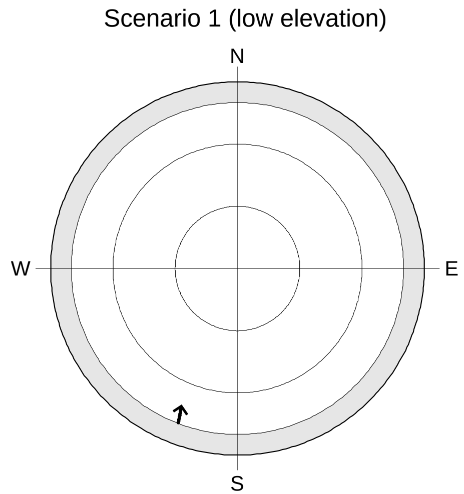
Purpose / Motivation:

- Dominant model parameters? ($\longrightarrow$ must be precisely known)
- Test of processing strategy
- Basis for analysis of real GNSS data in future

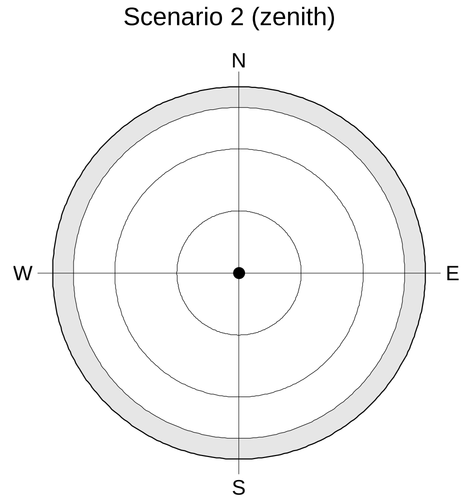
Simulations

Simulation scenarios and parameter sets

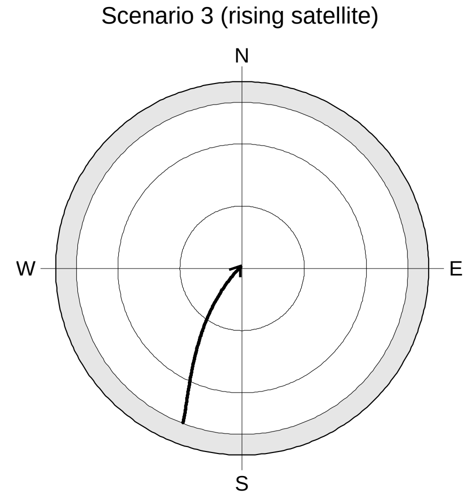
Scenarios:



100 observations
10 [sec] sampling
Elevation: 10 - 17 [deg]
Azimuth: 200 - 202 [deg]



100 observations
10 [sec] sampling
Elevation: 90 [deg]
Azimuth: 0 [deg]
(Fixed satellite)

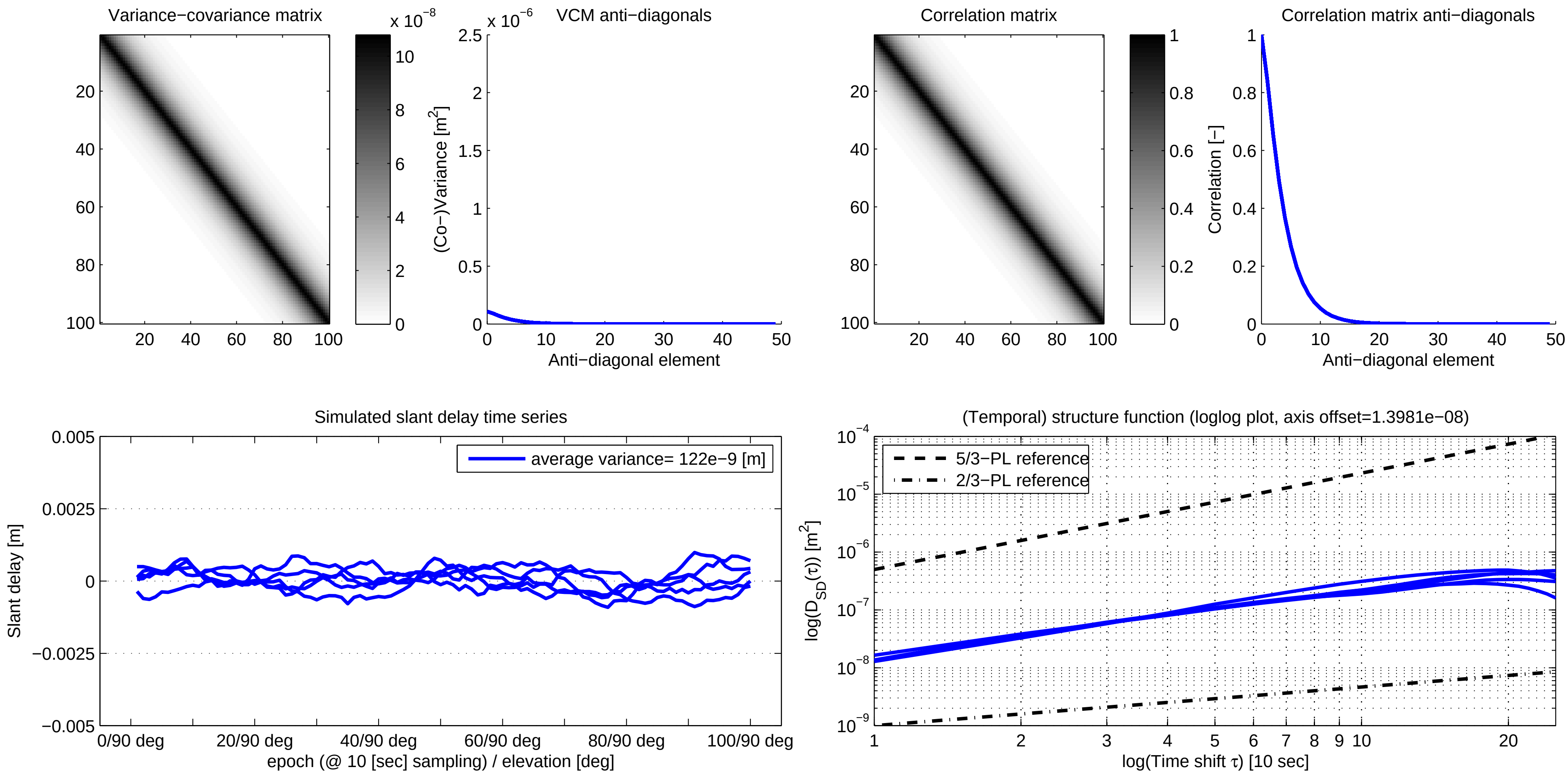


1000 observations
10 [sec] sampling
Elevation: 10 - 90 [deg]
Azimuth: 200 - 280 [deg]
(from real satellite)

Turbulence parameter variations:

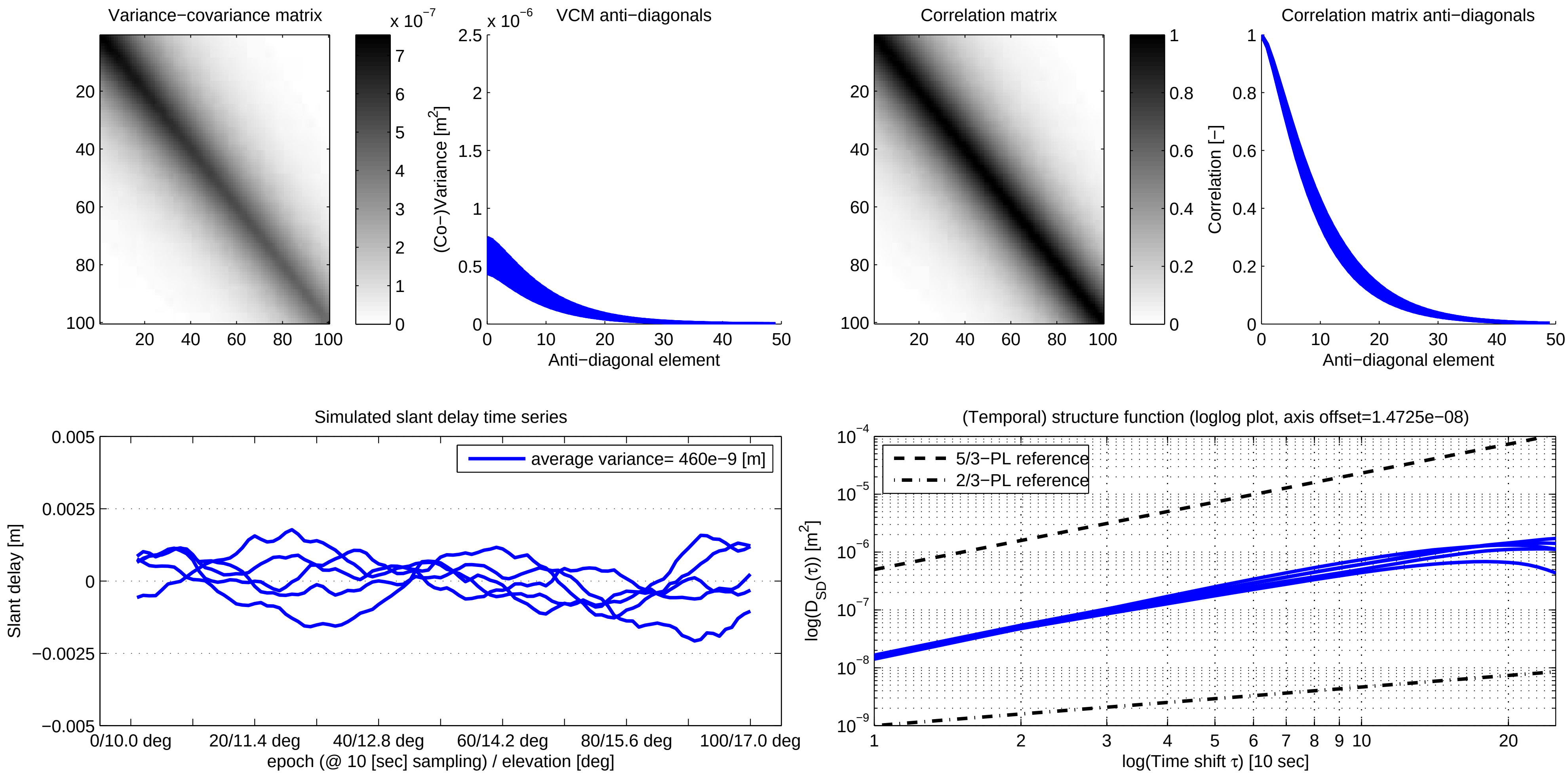
C_n^2 $[m^{-2/3}]$	L_0 $[m]$	v $[m/s]$	H $[m]$	α_v $[deg]$	Comment:
0.3×10^{-14}	3000	8	2000	0	Reference set
5.76×10^{-14}	6000	15	1000	90	
9.0×10^{-14}				180	
				270	

Scenario: zenith, parameter set: 5 (= average turbulence parameters):

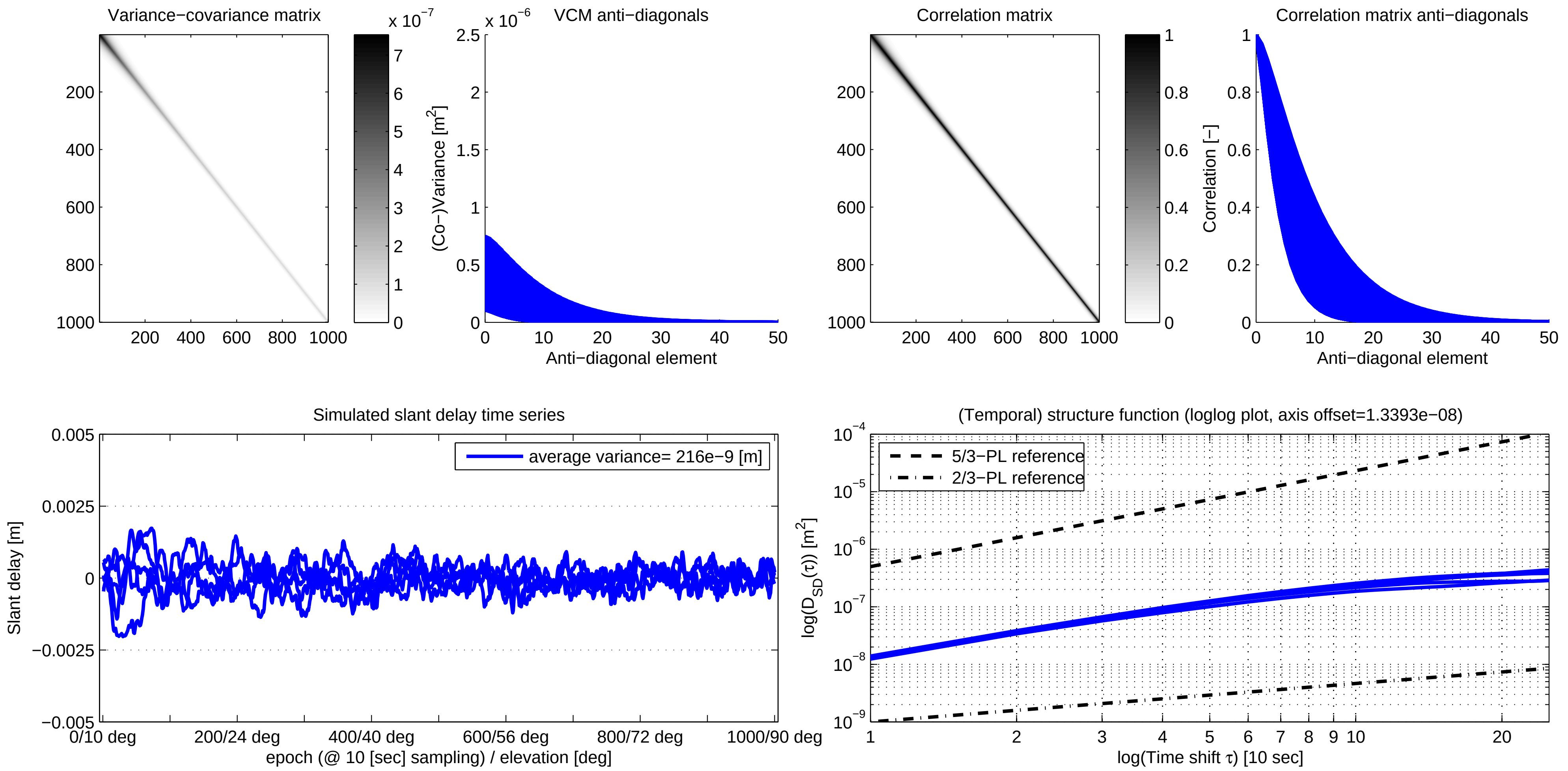


Simulations - low elevation

Scenario: low elevation, parameter set: 5 (= average turbulence parameters):

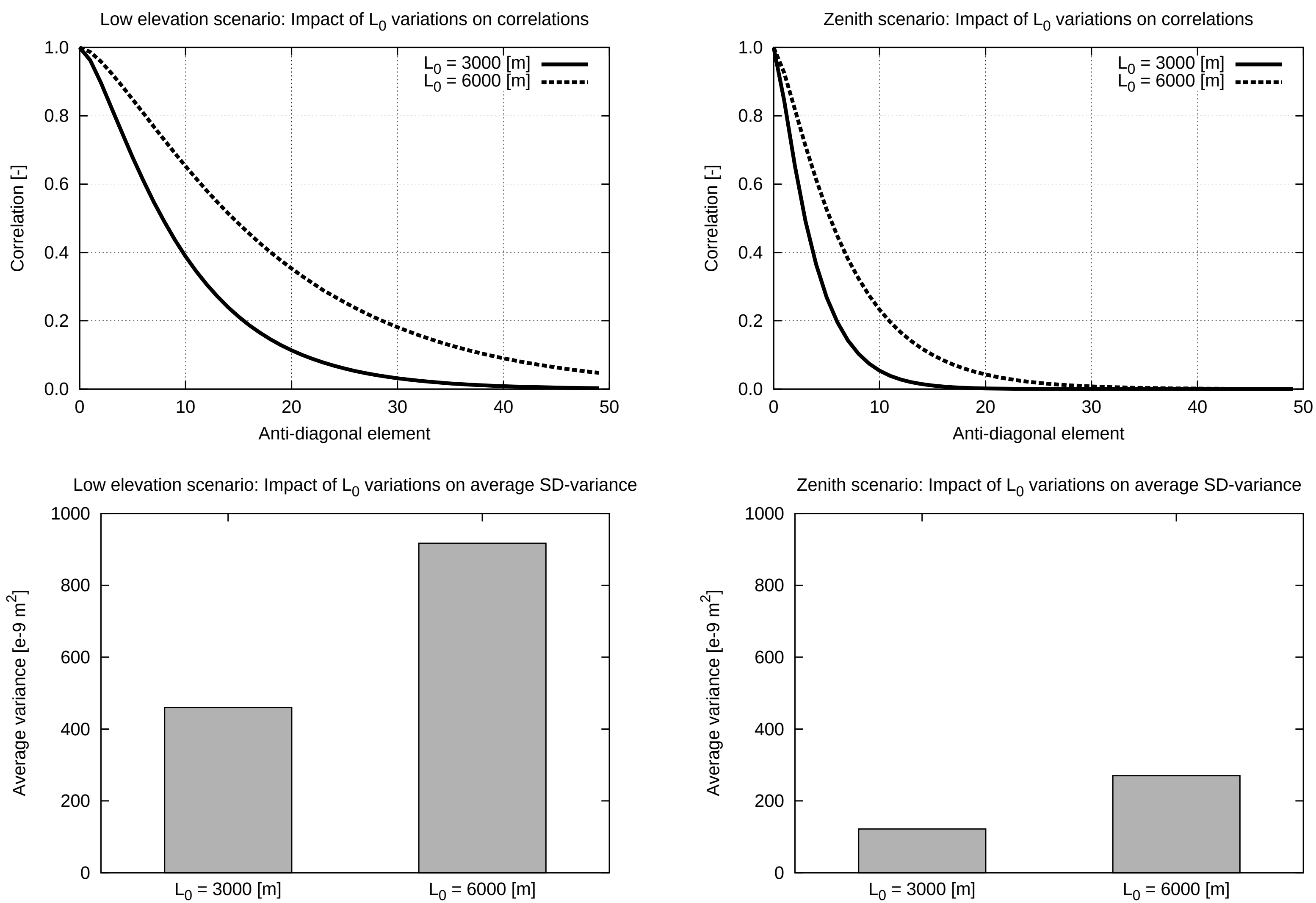


Scenario: rising satellite, parameter set: 5 (= average turbulence parameters):



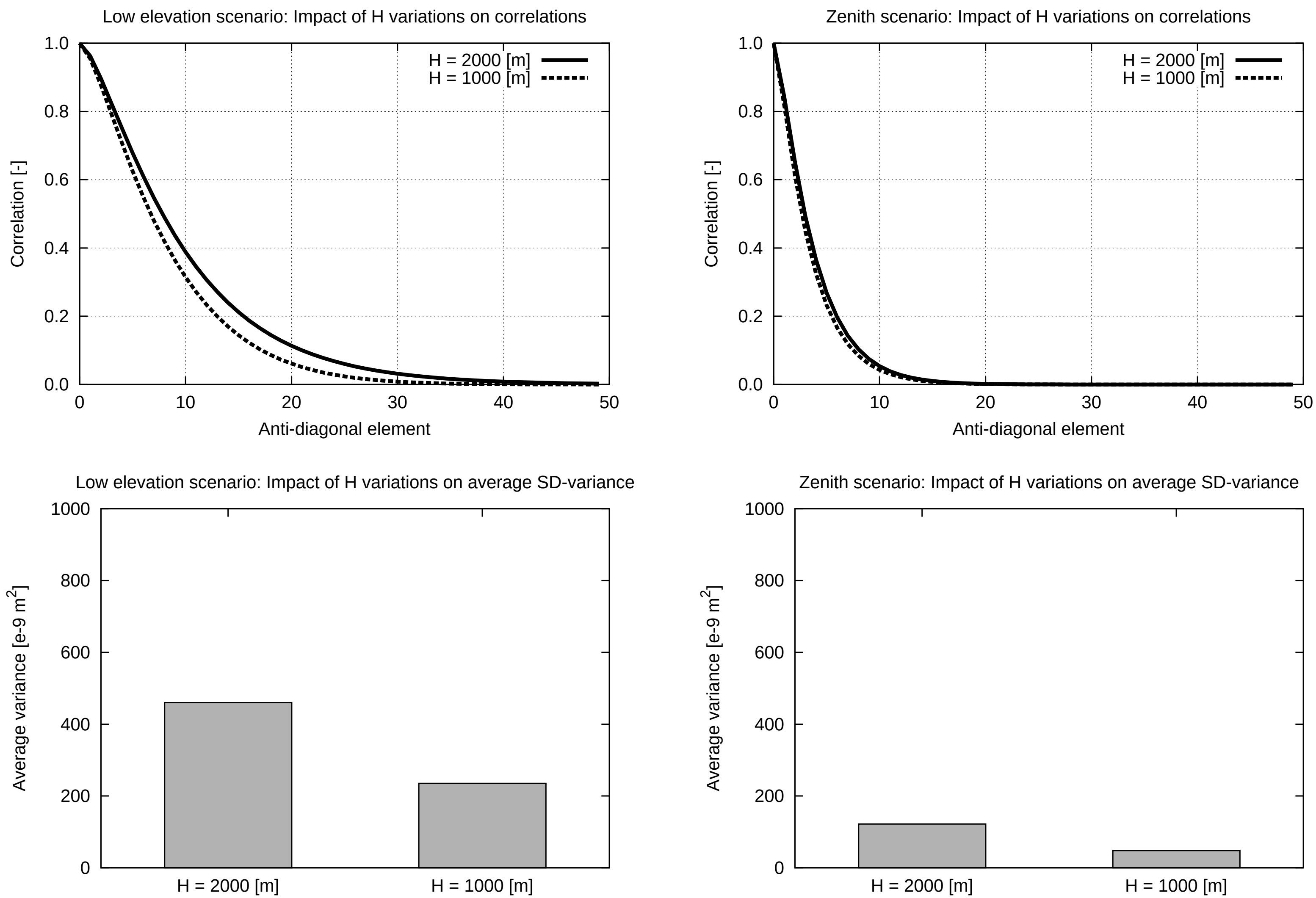
Impact of parameter variations

Impact of L_0 variations on mean correlations and average SD-variance:



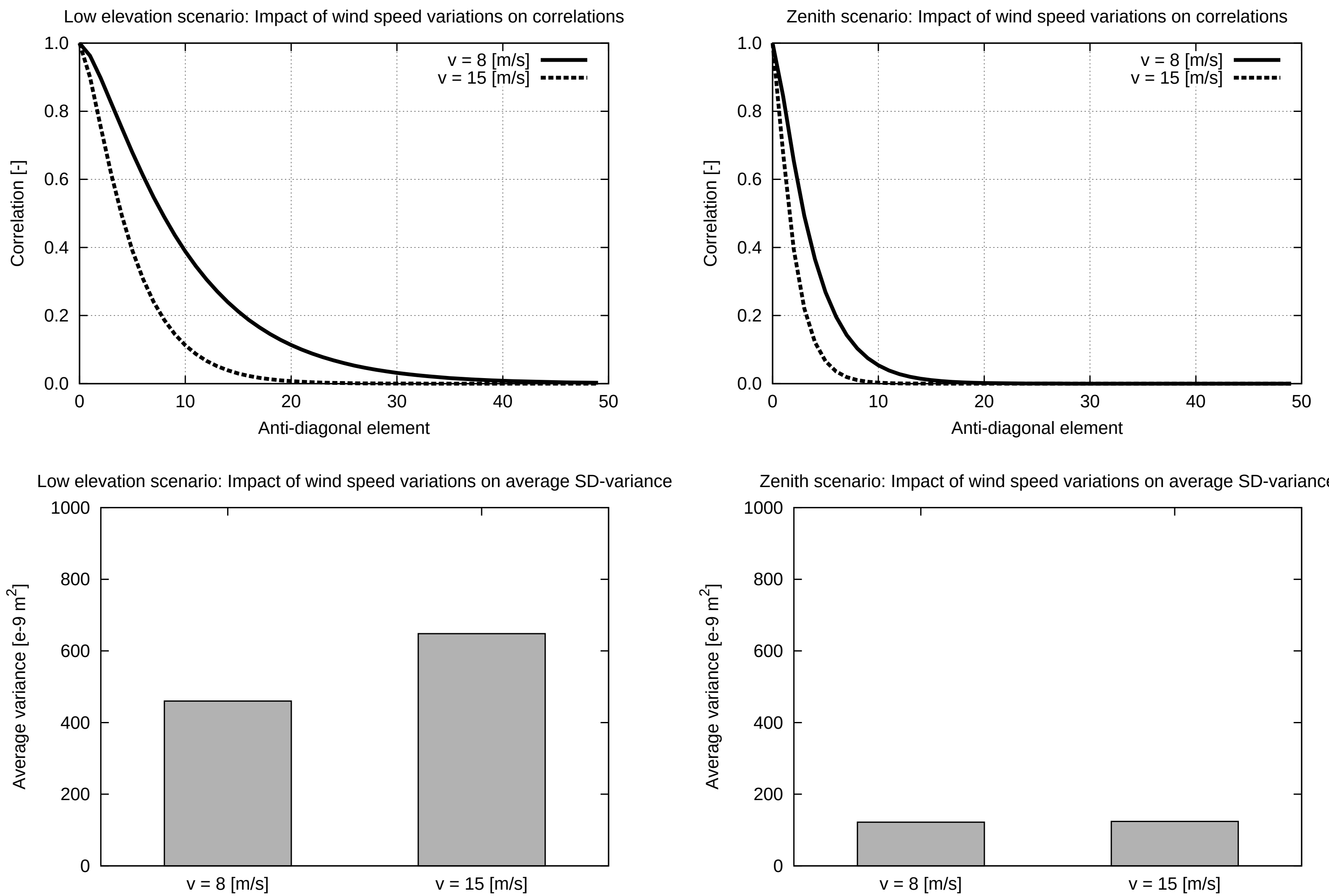
$\Rightarrow$ increasing $L_0 \rightarrow$ stronger turbulence, longer correlation time

Impact of H variations on mean correlations and average SD-variance:



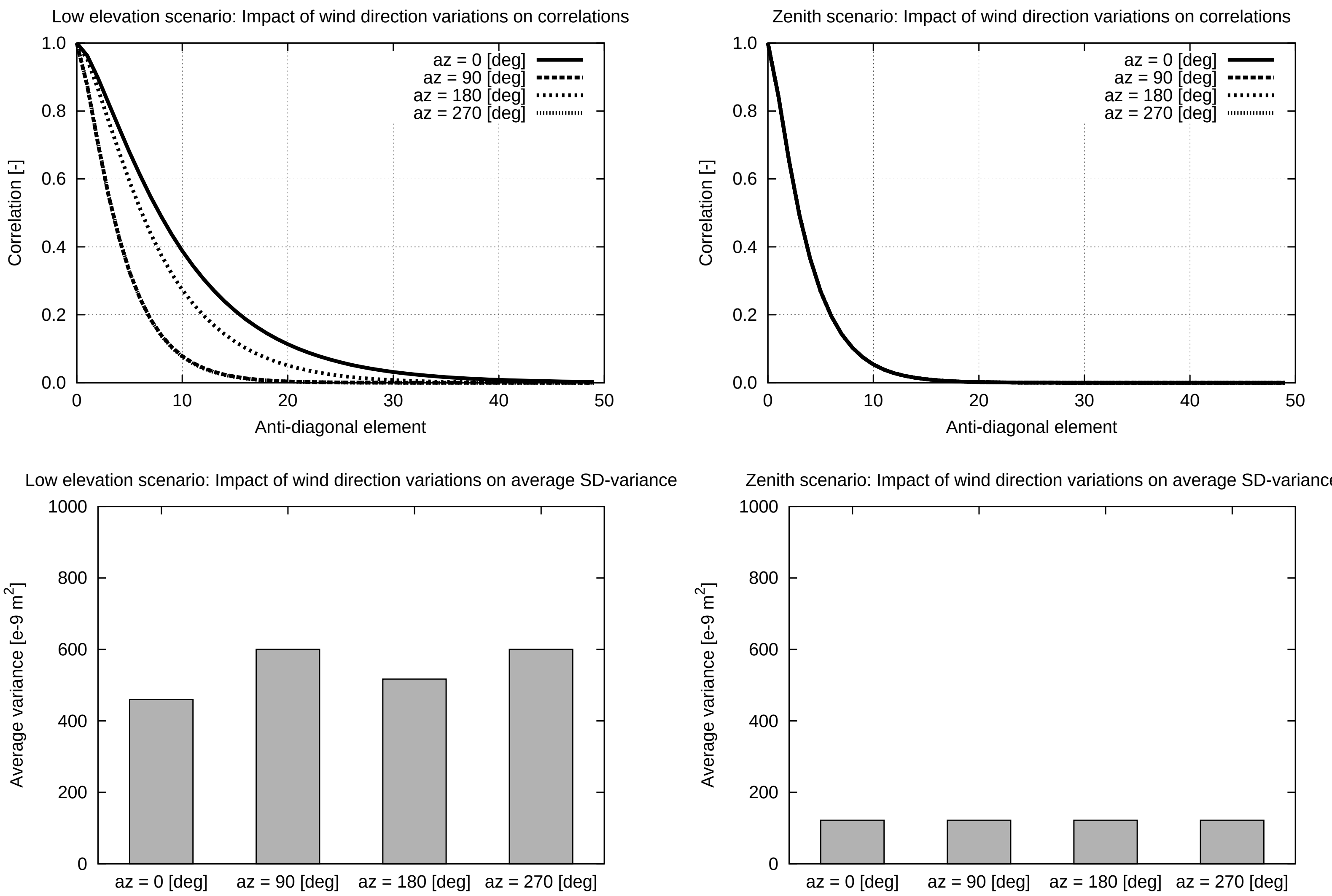
$\Rightarrow$ increasing H $\rightarrow$ stronger turbulence, small increase of correlations

Impact of wind speed variations on mean correlations and average SD-variance:



$\Rightarrow$ depending on geometry: increasing wind speed $\rightarrow$ decorrelating effect

Impact of wind direction variations on mean correlations and average SD-variance:



⇒ depending on geometry: decorrelating effect for orthogonal wind

Summary & Conclusions:

- generation of variance-covariance matrices of slant delays possible
- generation of slant delay variations possible
  - typical variations: $\pm$ 1 - 3 [mm]
  - correlation lengths: $\approx$ 200 [sec]
- simulated slant delay variations as expected:
  - 5/3 power law for all simulated time series
  - higher variations for low elevations
- superposition of geometric effects and atmospheric turbulence needs further investigation
- now: analysis of real GNSS data

Acknowledgements:

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